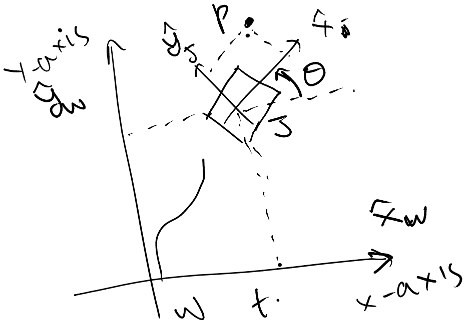
Coordinate transoformations (2D)



J = Jetbot

W = World

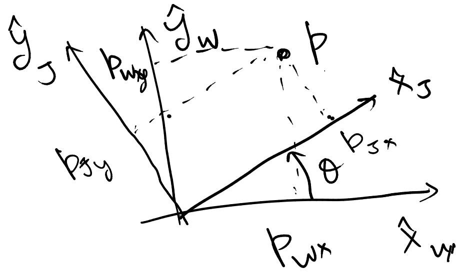
$p_w = ?$

θ - orientation

t_J - translation

$\underline{p_J}$ = coords of the point in Robot coordinate fram

① Rotation only ② translation only



Basis of a vector space

(p_{Jx}, p_{Jy}) known

(p_{wx}, p_{wy}) to find

coordinate frame

Basis of vector space

→ a set of vector

→ (1) linearly independent

→ (2) spans the vector space

(3) unit vectors

④ orthogonal to each other

$$p = \alpha \hat{x}_J + \beta \hat{y}_J$$

$$= \underbrace{\begin{bmatrix} \hat{x}_J & \hat{y}_J \end{bmatrix}}_{\substack{\text{Basis matrix} \\ B_J}} \underbrace{\begin{bmatrix} \alpha \\ \beta \end{bmatrix}}_{\text{coordinates}}$$

$$\left.\begin{aligned} \alpha &= p_{Jx} \\ \beta &= p_{Jy} \end{aligned}\right\} \quad \underline{p_J} = \begin{bmatrix} p_{Jx} \\ p_{Jy} \end{bmatrix}$$

$$\hat{x}_w = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \hat{y}_w = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$p = B_J \underline{p_J}$$

$$= B_w \underline{p_w}$$

$$B_w = \begin{bmatrix} \hat{x}_w & \hat{y}_w \end{bmatrix}$$

$$\underline{p_w} = ?$$

$$\boxed{\underline{p_w} = B_w^{-1} B_J \underline{p_J}}$$

$$\hat{x}_w^T \hat{x}_w = 1$$

$$\hat{x}_w \perp \hat{y}_w \quad \hat{x}_w^T \hat{y}_w = 0$$

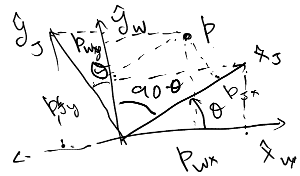
$$B_W^T B_W = \begin{bmatrix} \underset{2\times1}{\hat{x}_W} & \underset{2\times1}{\hat{y}_W} \end{bmatrix}_{2\times2}^T \begin{bmatrix} \hat{x}_W & \hat{y}_W \end{bmatrix}_{2\times2}$$

$$\boxed{B_W^T = B_W^{-1}}$$

$$= \begin{bmatrix} \hat{x}_W^T \\ \hat{y}_W^T \end{bmatrix} \begin{bmatrix} \hat{x}_W & \hat{y}_W \end{bmatrix}$$

$$= \begin{bmatrix} \hat{x}_W^T \hat{x}_W \nearrow 1 & \hat{x}_W^T \hat{y}_W \nearrow 0 \\ \hat{y}_W^T \hat{x}_W \nearrow 0 & \hat{y}^T \hat{y}_W \nearrow 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$\hat{x}_J = \_\_\_ \hat{x}_W + \_\ \hat{y}_W$$



$$= \cos\theta\ \hat{x}_W + \sin\theta\ \hat{y}_W$$

$$= \begin{bmatrix} \hat{x}_W & \hat{y}_W \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = B_W \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

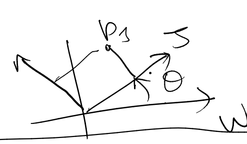
$$\hat{y}_J = -\sin\theta\ \hat{x}_W + \cos\theta\ \hat{y}_W = \underbrace{\begin{bmatrix} \hat{x}_W & \hat{y}_W \end{bmatrix}}_{B_W} \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

$$B_J = \begin{bmatrix} \hat{x}_J & \hat{y}_J \end{bmatrix} = \begin{bmatrix} B_W \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} & B_W \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix} \end{bmatrix}$$

$$= B_W \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}_{R(\theta)}$$

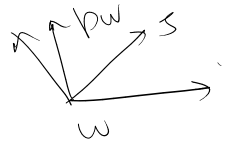
$$\underline{p_W} = B_W^{-1} B_J \underline{p_J} \qquad B_W^{-1} = B_W^T$$

① Rotation only $\quad B_J = B_W R(\theta)$



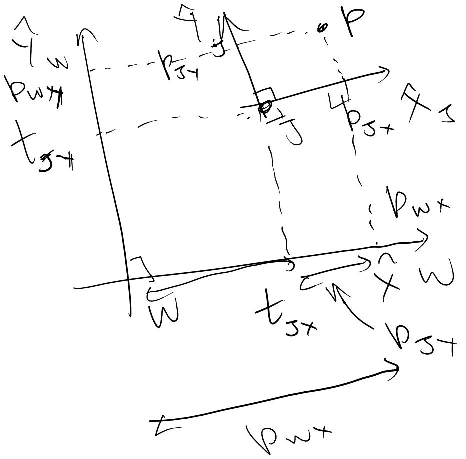
$$\boxed{\underline{p_W} = {}^{W}_{J}R(\theta)\ \underline{p_J} \qquad R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}$$

Excercise



$$\underline{p_J} = \underline{?}\ \underline{p_W}$$

② translation only part



$$p_{Wx} = p_{Jx} + t_{Jx}$$

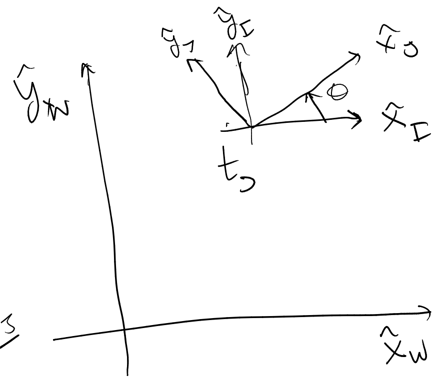
$$p_{Wy} = p_{Jy} + t_{Jy}$$

$$\underline{p_W} = \underline{p_J} + \underline{t_J}$$

(3) Rot + trans

J = Jetbot
I = Intermediate
W = World

$$J \xrightarrow{Rot} I \xrightarrow{trans} W$$



$$\underline{p_I} = {}^I R_J(\theta)\, \underline{p_J}$$

$$\underline{p_W} = \underline{p_I} + \underline{t_J}$$ translation only

$$\boxed{\underline{p_W} = {}^W R_J(\theta)\, \underline{p_J} + {}^W t_J}$$

(4) Transformation matrix represent

Homogeneous coordinate

$$\begin{bmatrix} \underline{p_W} \\ 1 \end{bmatrix}_{3\times1} = \underbrace{\begin{bmatrix} {}^W R_J(\theta)_{2\times2} & {}^W t_{J\,2\times1} \\ \mathbf{0}^T_{1\times2} & 1 \end{bmatrix}_{3\times3}}_{\text{Transformation matrix } {}^W T_J} \begin{bmatrix} \underline{p_J} \\ 1 \end{bmatrix}_{3\times1}$$