ECE275 Final exam Fall 2022

Instructor: Vikas Dhiman (vikas.dhiman@maine.edu)

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Student Name: Student Email:

1 Instructions

- There are seven questions. First two questions (Prob 1 and Prob 2) are mandatory. **Do any four out of the five remaining problems**. For extra credit, do all seven.
- Maximum number of marks is 120 (140 with extra-credit). This exam amounts 10% toward the final grade.
- Time allowed is 120 minutes.
- In order to minimize distraction to your fellow students, you may not leave during the last 10 minutes of the examination.
- The examination is closed-book. One 8×11 in two-sided cheatsheet is allowed.
- Non-programmable calculators are permitted.
- Please use a pen or heavy pencil to ensure legibility. Colored pens/pencils are recommended for K-map grouping.
- Please show your work; where appropriate, marks will be awarded for proper and well-reasoned explanations.

Problem 1. *A sequential circuit is to be used to control the operation of a vending machine which dispenses a \$0.25 product. The circuit has three inputs (N, D, and Q) and two outputs (R and C). The coin detector mechanism in the vending machine is synchronized with the same clock as the sequential circuit you are to design. The coin detector outputs a single 1 to the N, D, or Q input for every nickel (5 cents), dime (10 cents), or quarter (25 cents), respectively, that the customer inserts. Only one input will be 1 at a time. When the customer has inserted at least \$0.25 in any combination of nickels, dimes, and quarters, the vending machine must give change and dispense the product. The coin return mechanism gives change by returning nickels to the customer. For every 1 output on C, the coin return mechanism will return one nickel to the customer. The product is dispensed when the circuit outputs a single 1 on output R. The circuit should reset after dispensing the product.*

Example: *The customer inserts a nickel, a dime, and a quarter. The circuit inputs and outputs could look like this:*

Inputs:

	Money	0	0	0	5	5	5	5	15	15	15	40	35	30	25	25	0	0
Inputs	N=	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	D=	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
	Q=	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
Outputs	R=	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
	C=	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0

Note that any number of 0's can occur between 1 inputs. Derive a Moore state table for the sequential circuit, and for each state indicate how much money the customer has inserted or how much change is due. (30 marks)

Moore state table

Money inserted	Change due	Present State PS	Next State NS: NDQ = 000	100 (Nickel)	010 (Dime)	001 (Quarter)	Output (RC)
0	0	S_0	S_0	S_1	S_2	S_3	0 0
5	0	S_1	S_1	S_2	S_4	S_5	0 0
10	0	S_2	S_2	S_4	S_6	S_7	0 0
25	0	S_3	S_0	S_0	S_0	S_0	1 0
15	0	S_4	S_4	S_6	S_3	S_8	0 0
30	5	S_5	S_3	S_3	S_3	S_3	0 1
20	0	S_6	S_6	S_3	S_5	S_9	0 0
35	10	S_7	S_5	S_5	S_5	S_5	0 1
40	15	S_8	S_7	S_7	S_7	S_7	0 1
45	20	S_9	S_8	S_8	S_8	S_8	0 1

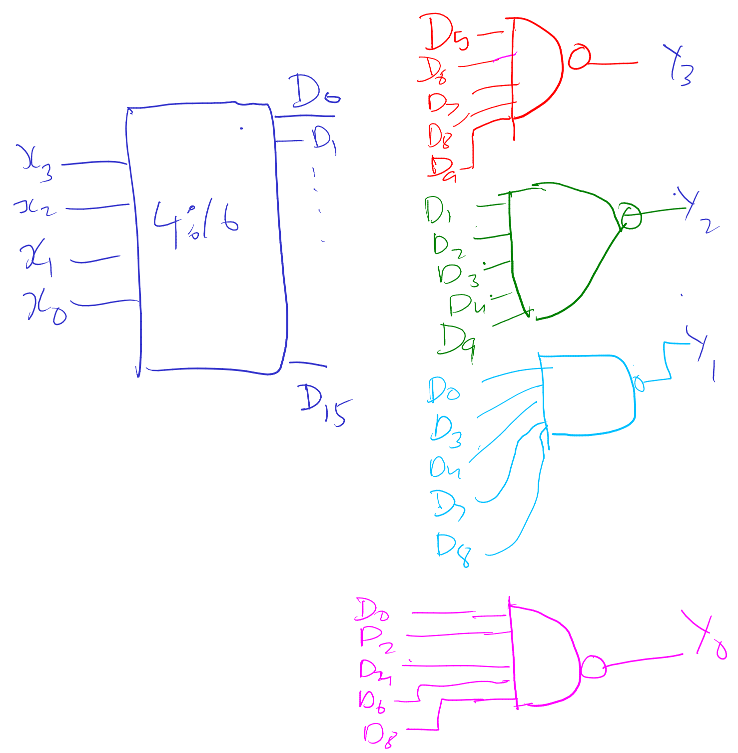
0 0000
⋮
9 1001

Problem 2. *Realize a BCD to excess-3 code converter using a 4-to-16 decoder with active low outputs and a minimum number of gates. (The excess-3 code is obtained from the binary numbers (0-9) by adding 3 (0011) to each of the binary numbers.) (Active low outputs mean that the selected output pin is 0, while all others are 1.) (10 marks)*

$Y_3 = \overline{D_5 \cdot D_6 \cdot D_7 \cdot D_8 \cdot D_9}$

$Y_3 = \overline{D_5} + \overline{D_6} + \overline{D_7} + \overline{D_8} + \overline{D_9}$

Count	BCD x_3	x_2	x_1	x_0	Excess-3 Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	1	1
1	0	0	0	1	0	1	0	0
2	0	0	1	0	0	1	0	1
3	0	0	1	1	0	1	1	0
4	0	1	0	0	0	1	1	1
5	0	1	0	1	1	0	0	0
6	0	1	1	0	1	0	0	1
7	0	1	1	1	1	0	1	0
8	1	0	0	0	1	0	1	1
9	1	0	0	1	1	1	0	0



20min x 2
40 min

Problem 3. *Reduce the following state table to a minimum number of states using implication charts (20 marks).*

Present State	Next State X = 0	Next State 1	Output Z
A	A	B	1
B	C	E	0
C	F	G	1
D	C	A	0
E	~~I~~ B	G	1
F	~~H~~ F	~~I~~ B	1
G	C	F	0
~~H~~	~~F~~	~~B~~	~~1~~
~~I~~ → B	~~C~~	~~E~~	~~0~~

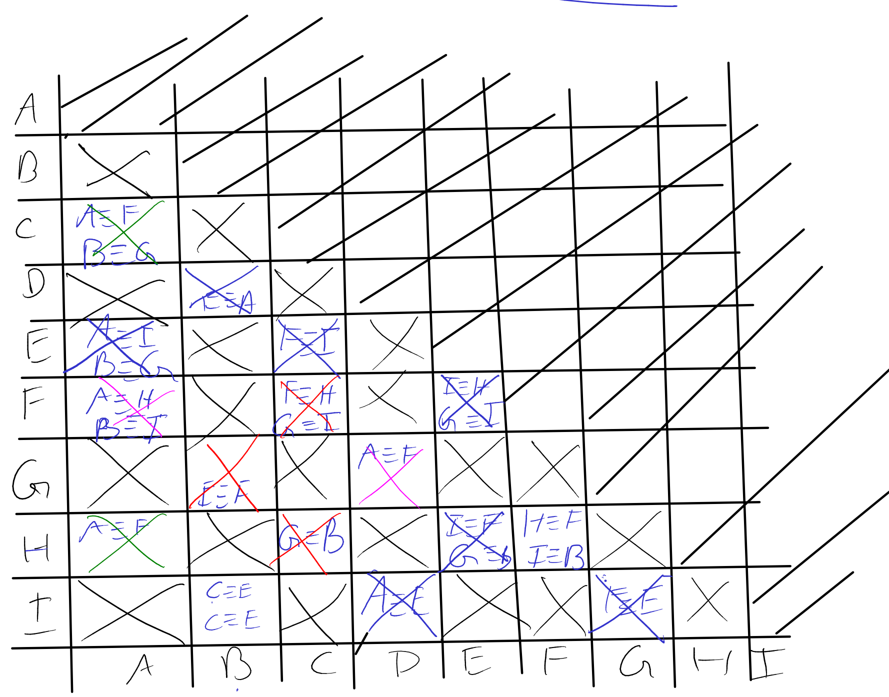
X due to output difference

X due to Round 1

X due to Round 2

X due to Round 3

X due to Round 4



B ≡ I → B

F ≡ H → F

Problem 4. 1. *For the following state table, use the three guidelines to determine which of the three possible nonequivalent state assignments should give the best solution (20 marks).*

2. *Using your answer to (a), derive J-K flip-flop input equations and the output equations (20 marks).*

(40 min)

X_2 (80 min)

Present State	Next state $X_1X_2 = 00$	01	11	10	Outputs Z_1Z_2 $X_1X_2 = 00$	01	11	10
A	A	A	C	C	01	01	01	01
B	B	D	B	D	11	11	11	11
C	A	A	B	D	11	11	00	00
D	D	B	A	C	01	01	01	01

Guidelines

Highest priority: (A,C)², (B,C)², (A,D)

Medium priority: (A,C), (B,D), (A,B,D), (A,B,C,D)

Lowest priority: 00/01 (A,D); 00/11 (B,C)

4 states 2 ff

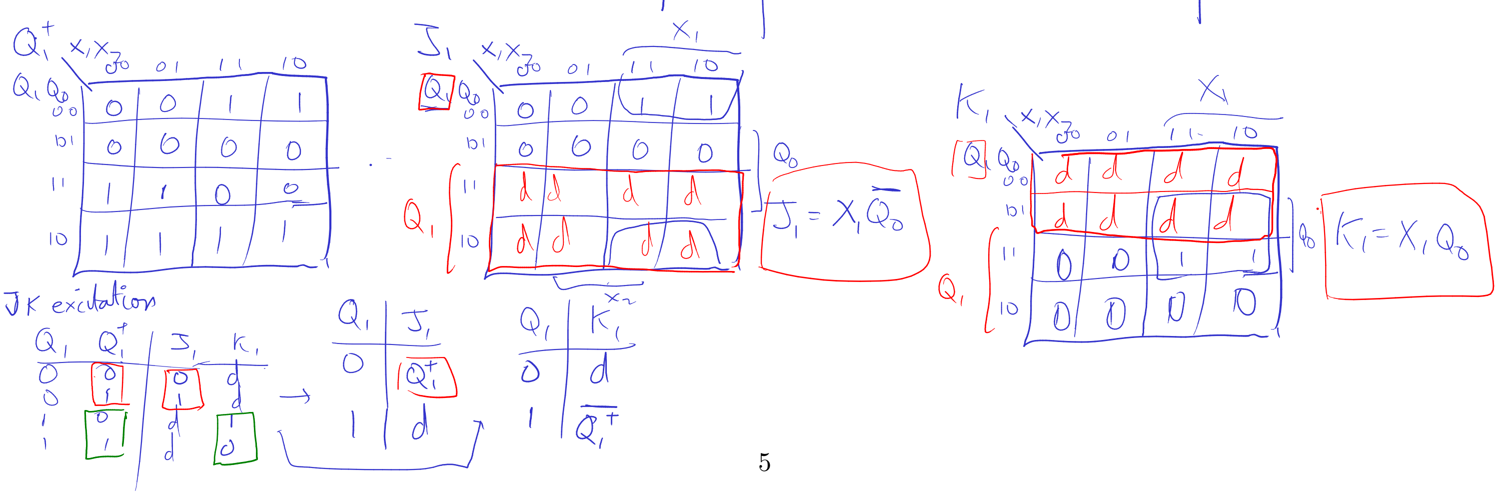
$2^1 < 4 \le 2^2$

State map

State	State assignment Q_1Q_0
A	01
B	10
C	00
D	11

Present State	Next state $X_1X_2 = 00$	01	11	10	Outputs Z_1Z_2 $X_1X_2 = 00$	01	11	10
A	A	A	C	C	01	01	01	01
B	B	D	B	D	11	11	11	11
C	A	A	B	D	11	11	00	00
D	D	B	A	C	01	01	01	01

	PS Q_1Q_0	NS $X_1X_2 = 00$	01	11	10	Outputs (Z_1Z_2) $X_1X_2 = 00$	01	11	10
C	00	01	01	10	11	11	11	00	00
A	01	01	01	00	00	01	01	01	01
D	11	11	10	01	00	01	01	01	01
B	10	10	11	10	11	11	11	11	11



$J_1 = X_1\bar{Q_0}$

$K_1 = X_1 Q_0$

Q_0^+

Q_1Q_0 \ x_1x_2	00	01	11	10
00	1	1	0	1
01	1	1	0	0
11	1	0	1	0
10	0	1	0	1

J_0

Q_1Q_0 \ x_1x_2	00	01	11	10
00	1	1	0	1
01	d	d	d	d
11	d	d	d	d
10	0	1	0	1

$$J_0 = x_1\bar{x}_2 + \bar{x}_1 x_2 + \bar{x}_1 \bar{Q}_1$$

$$\bar{J}_0 = x_1 x_2 + \bar{x}_1 \bar{x}_2 Q_1$$

$$J_0 = (\bar{x}_1 + \bar{x}_2)(x_1 + x_2 + \bar{Q}_1)$$

K_0

Q_1Q_0 \ x_1x_2	00	01	11	10
00	d	d	d	d
01	0	0	1	1
11	0	1	0	1
10	d	d	d	d

$$K_0 = x_1\bar{Q}_1 + x_1\bar{x}_2 + \bar{x}_1 x_2 Q_1$$

Q_0	K_0
0	d
1	$\bar{Q}_0^+$

Q_0	J_0
0	Q_0^+
1	d

Z_1

Q_1Q_0 \ x_1x_2	00	01	11	10
00	1	1	0	0
01	0	0	0	0
11	0	0	0	0
10	1	1	1	1

$$Z_1 = Q_1\bar{Q}_0 + \bar{x}_1\bar{Q}_0$$

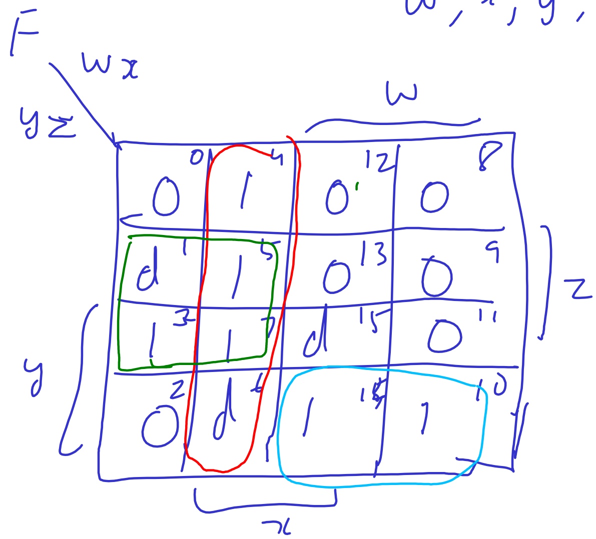
Z_2

Q_1Q_0 \ x_1x_2	00	01	11	10
00	1	1	0	0
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

$$Z_2 = \bar{x}_1 + Q_0 + Q_1$$

Problem 5. *Use a 4-to-1 multiplexer and a minimum number of external gates to realize the function* $F(w,x,y,z) = \sum m(3,4,5,7,10,14) + \sum d(1,6,15)$. *The inputs are only available uncomplemented (20 marks).*

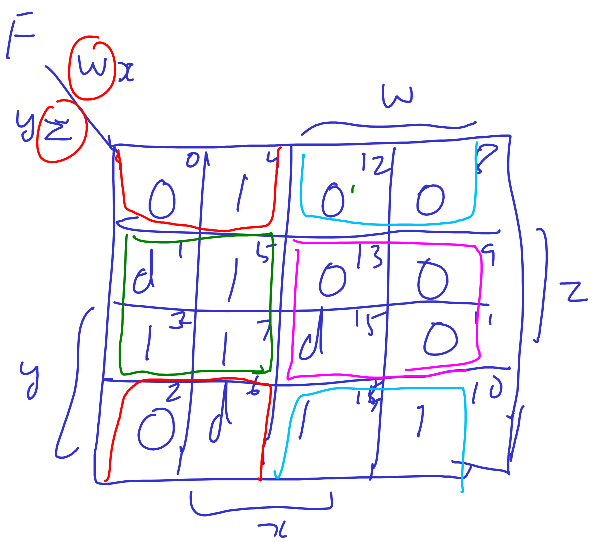
$\bar{w}, \bar{x}, \bar{y}, \bar{z}$

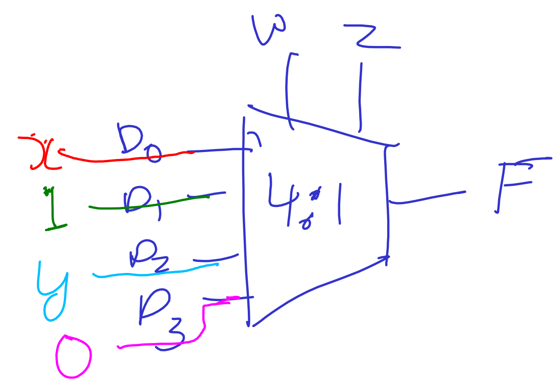


$F = \bar{w}x + \bar{w}z + wy\bar{z}$

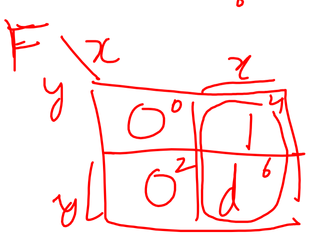
w occurs 3 times
x occurs 1 times
y " 1 "
z " 2 times

Pick w, z as select bits





when wz = 00
$F = D_0 = ?$

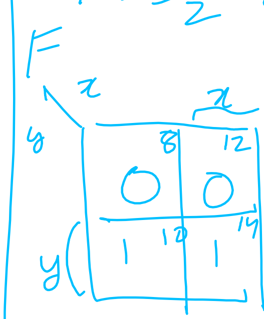


$F = D_0 = x$

when wz = 01
$F = D_1 = ?$

$F = D_1 = 1$

when wz = 10
$F = D_2 = ?$

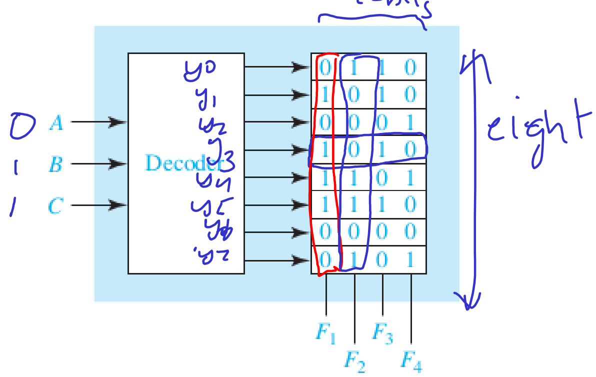


$F = D_2 = y$

when wz = 11
$F = D_3 = ?$

$F = D_3 = 0$

Problem 6. *The following diagram shows the pattern of 0's and 1's stored in a ROM with eight words and four bits per word. What will be the values of* $F_1, F_2, F_3,$ *and* F_4 *if* $A = 0$ *and* $B = C = 1$*? Also give the minterm expansions for* F_1 *and* F_2 *(20 marks).*



A	B	C	y_0	y_1	… y_3	y_7
0	0	0	1	0	- - - -	0
0	0	1	0	1	- - -	0
0	1	1	0	0	1	0

when $ABC = 011$

then $F_1 F_2 F_3 F_4 = 1010$ — $F_1 = 1$, $F_2 = 0$, $F_3 = 1$, $F_4 = 0$

minterm expansion for F_1

Recall for a Decoder $y_0 = \bar{A}\bar{B}\bar{C} = m_0$

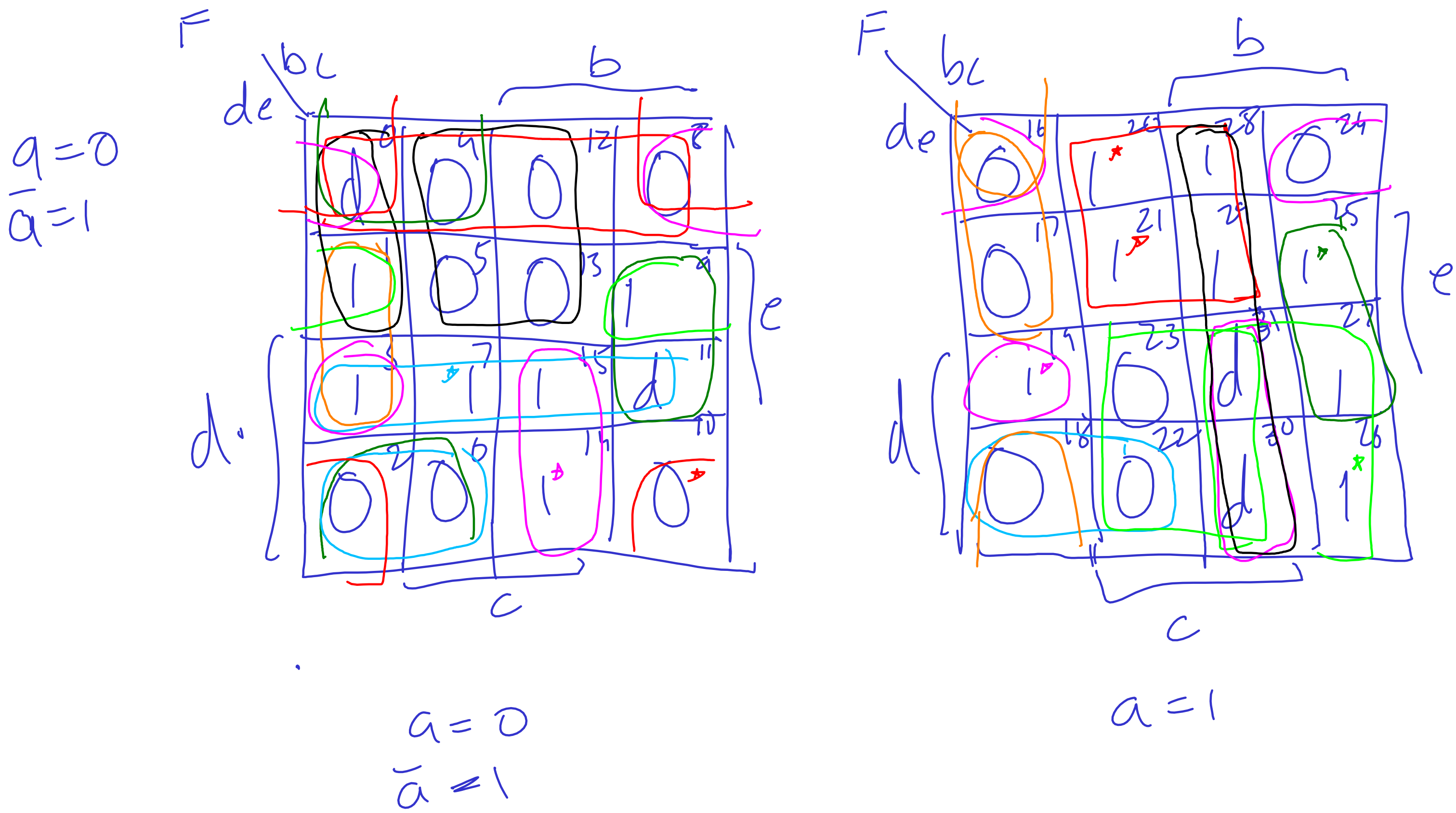
$y_1 = \bar{A}\bar{B}C = m_1$

$F_1 = m_1 + m_3 + m_4 + m_5 = \sum m(1,3,4,5)$

$F_2 = m_0 + m_4 + m_5 + m_7 = \sum m(0,4,5,7)$

Problem 7. *Find the minimum cost circuit for the following function using K-map. Find both sum-of-products and product-of-sum forms and find the minimum cost one.*

$F(a,b,c,d,e) = \prod M(2,4,5,6,8,10,12,13,16,17,18,22,23,24) + \prod D(0,11,30,31)$ *(20 marks).*



At Maxterms F=0

SOP form

$F = bcd + \bar{a}de + \bar{a}\bar{c}\bar{d}e + b\bar{c}e + ac\bar{d} + \bar{b}\bar{c}de + abd$

Cost = 7+1+3×5+4×2+7 = 38

POS form

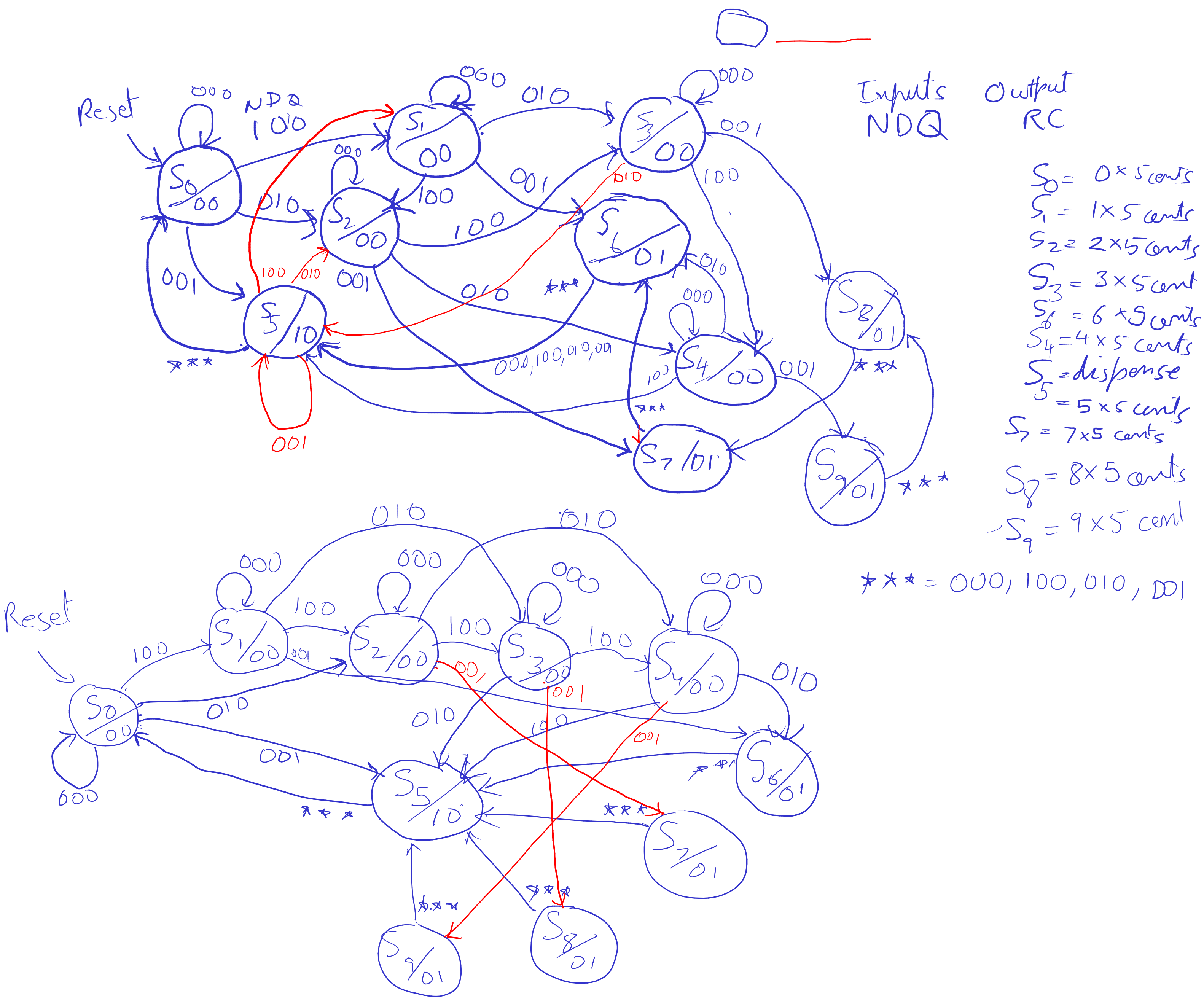
$\bar{F} = \bar{a}\bar{c}\bar{e} + \bar{a}c\bar{d} + \bar{b}d\bar{e} + acd + \bar{c}d\bar{e} + a\bar{b}\bar{c}d$

$F = (a+c+e)(a+\bar{c}+d)(b+\bar{d}+e)(\bar{a}+\bar{c}+\bar{d})(c+d+e)(\bar{a}+b+c+d)$

Cost = 6+1+3×5+4×1+6 = 32

POS is minimum cost

Problem 1 (Alternative solution)



Money inserted (cents)	Change due (cents)	PS	NS Inputs (NDQ) 000	100	010	001	output (RC)
0	0	S_0	S_0	S_1	S_2	S_5	0 0
5	0	S_1	S_1	S_2	S_3	S_6	0 0
10	0	S_2	S_2	S_3	S_4	S_7	0 0
15	0	S_3	S_3	S_4	S_5	S_8	0 0
20	0	S_4	S_4	S_5	S_6	S_9	0 0
25	0	S_5	S_0	S_0	S_0	S_0	1 0
30	5	S_6	S_5	S_5	S_5	S_5	0 1
35	10	S_7	S_6	S_6	S_6	S_6	0 1
40	15	S_8	S_7	S_7	S_7	S_7	0 1
45	20	S_9	S_8	S_8	S_8	S_8	0 1