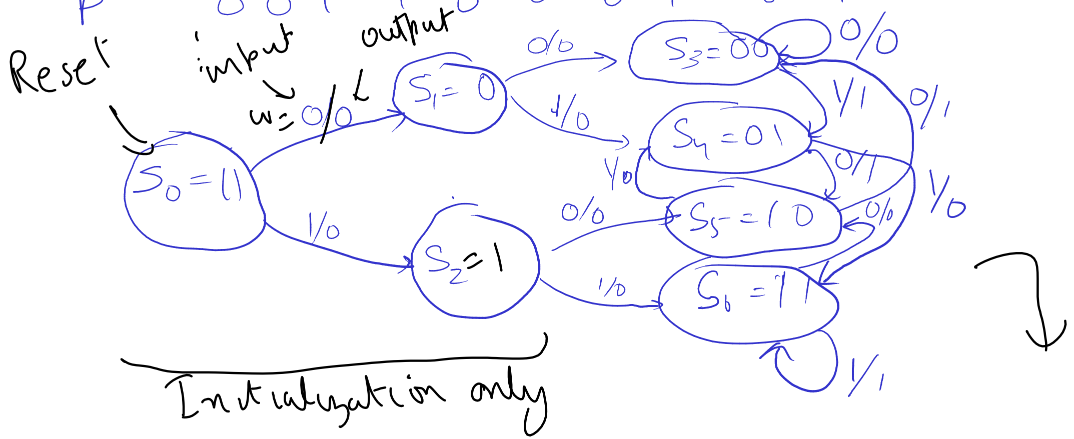
Practice problems

Problem 1

Example sequence (Mealy)

t	0	1	2	3	4	5	6	7	8	9
w	0	0	1	0	1	1	0	0	0	1
p	0	0	1	1	0	0	0	1	0	1



State table

Seq	Present state	Next State w=0	Next State w=1	Output (p) w=0	Output (p) w=1
""	S_0	S_1	S_2	0	0
"0"	S_1	S_3	S_4	0	0
"1"	S_2	S_5	S_6	0	0
"00"	S_3	S_3	S_4	0	1
"01"	S_4	S_5	S_6	1	0
"10"	S_5	S_3	S_4	1	0
"11"	S_6	S_5	S_6	0	1

Problem 1 Alternative solution

Example sequence

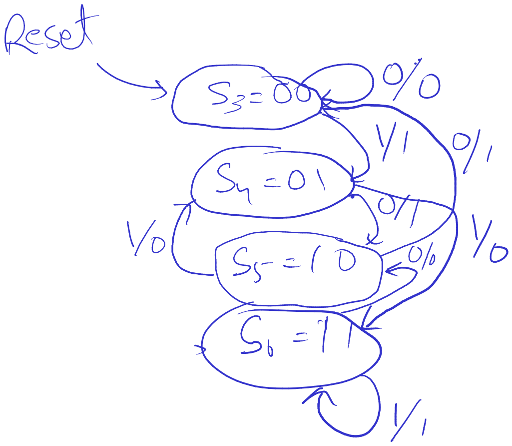
Reset

Assume that inputs before reset were '00'

time →

Input w = 00 : 0 1 0 1 1 0 0 0 1

Output p = : 0 1 0 0 0 0 1 0 1



By making a different assumption, we
are able to remove the 3 initialization states.

State table

eq	Present state	Next State		Output (p)	
		w=0	w=1	w=0	w=1
"00"	S_3	S_3	S_4	0	1
"01"	S_4	S_5	S_6	1	0
"10"	S_5	S_3	S_4	1	0
"11"	S_6	S_5	S_6	0	1

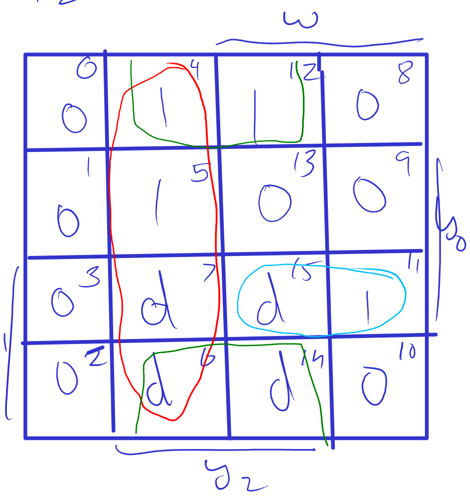
Problem 2

Moore modulo 6 counter

State assigned table (Not optimal)

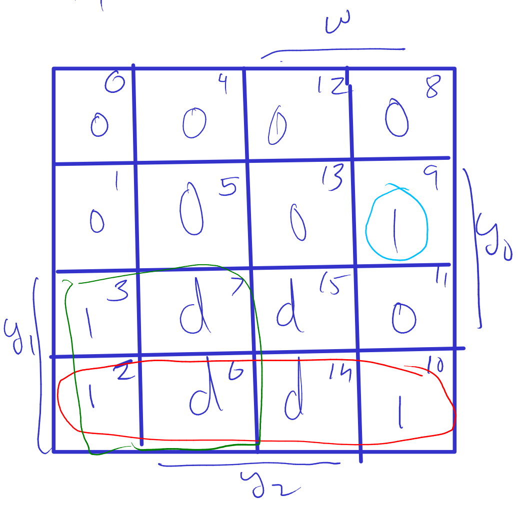
Present State	Next State		Output
	w = 0	w = 1	
y_2 y_1 y_0	Y_2 Y_1 Y_0	Y_2 Y_1 Y_0	$Z_2 = y_2$, $Z_1 = y_1$, $Z_0 = y_0$
0 0 0	0 0 0	0 0 1	
0 0 1	0 0 1	0 1 0	
0 1 0	0 1 0	0 1 1	
0 1 1	0 1 1	1 0 0	
1 0 0	1 0 0	1 0 1	
1 0 1	1 0 1	0 0 0	
1 1 0	d d d	d d d	
1 1 1	d d d	d d d	

Y_2



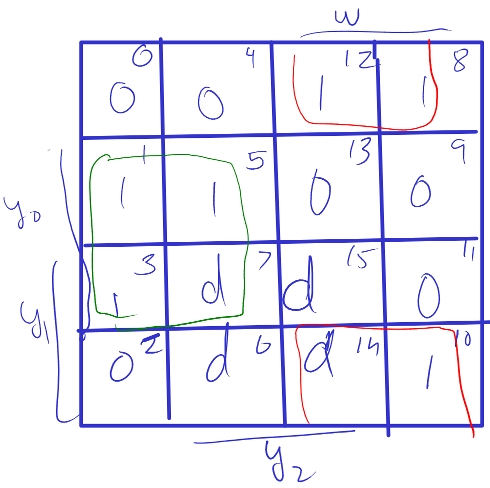
^{0} 0	^{4} 1	^{12} 1	^{8} 0
^{1} 0	^{5} 1	^{13} 0	^{9} 0
^{3} 0	^{7} d	^{15} d	^{11} 1
^{2} 0	^{6} d	^{14} d	^{10} 0

Y_1



^{0} 0	^{4} 0	^{12} 0	^{8} 0
^{1} 0	^{5} 0	^{13} 0	^{9} 1
^{3} 1	^{7} d	^{15} d	^{11} 0
^{2} 1	^{6} d	^{14} d	^{10} 1

Y_0

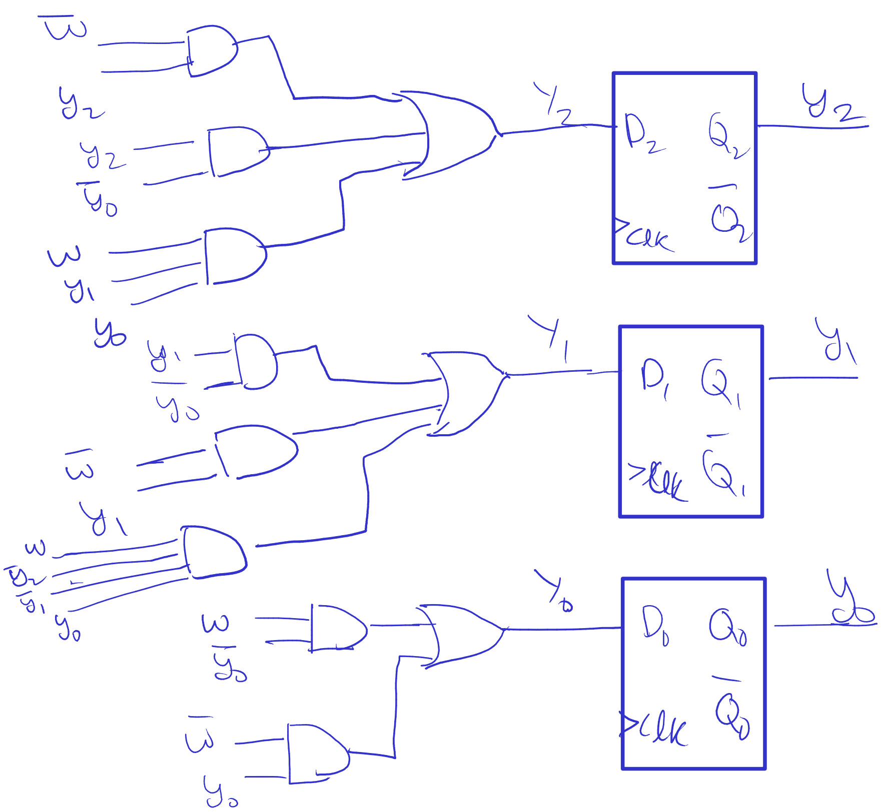


^{0} 0	^{4} 0	^{12} 1	^{8} 1
^{1} 1	^{5} 1	^{13} 0	^{9} 0
^{3} 1	^{7} d	^{15} d	^{11} 0
^{2} 0	^{6} d	^{14} d	^{10} 1

$$Y_2 = \bar{w} y_2 + y_2 \bar{y}_0 + w y_1 y_0$$

$$Y_1 = y_1 \bar{y}_0 + \bar{w} y_1 + w \bar{y}_2 \bar{y}_1 y_0$$

$$Y_0 = w \bar{y}_0 + \bar{w} y_0$$



W̄
y2
y2
ȳ0
w
y1
y0
Y2
D2
Q2
Q̄2
>clk
y2
y1
ȳ0
W̄
y1
w
ȳ2
ȳ1
y0
Y1
D1
Q1
Q̄1
>clk
y1
w
ȳ0
W̄
y0
Y0
D0
Q0
Q̄0
>clk
y0

Problem 3   3-bit counter like circuit (Moore)

State assigned table

Present State	Next State		Output
	$w=0$	$w=1$	
$y_2\ y_1\ y_0$	$Y_2^+ Y_1^+ Y_0^+$	$Y_2^+ Y_1^+ Y_0^+$	$Z_2=y_2$ $Z_1=y_1$ $Z_0=y_0$
0 0 0	1 1 1	0 1 0	
0 0 1	0 0 0	0 1 1	
0 1 0	0 0 1	1 0 0	
0 1 1	0 1 0	1 0 1	
1 0 0	0 1 1	1 1 0	
1 0 1	1 0 0	1 1 1	
1 1 0	1 0 1	0 0 0	
1 1 1	1 1 0	0 0 1	

How to convert Y_2+ K-map to J_2 and K_2 K-map?

J-K excitation table

y_2	Y_2^+	J_2	K_2	
0	0	0	d	(hold or reset)
0	1	1	d	(toggle on set)
1	0	d	1	(toggle on reset
1	1	d	0	(hold on set)

y_2	Y_2^+	J_2
0	0	0
0	1	1
1	0	d
1	1	d

when $y_2=0$ $J_2=Y_2^+$

when $y_2=1$ then $J_2=d$

y_2	J_2
0	Y_2^+
1	d

y_2	Y_2^+	K_2
0	0	d
0	1	d
1	0	1
1	1	0

when $y_2=0$ then $K_2=d$

when $y_2=1$ then $K_2=\overline{Y_2^+}$

y_2	K_2
0	d
1	$\overline{Y_2^+}$

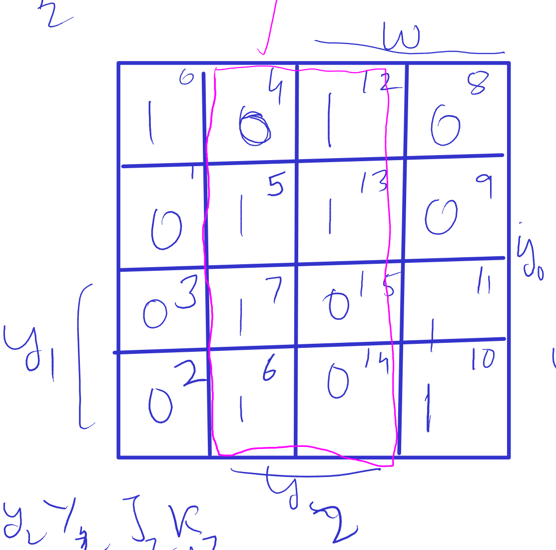
where $y_2=1$ put $J_2=d$

where $y_2=0$ put $J_2=Y_2^+$

where $y_2=0$ put $K_2=d$

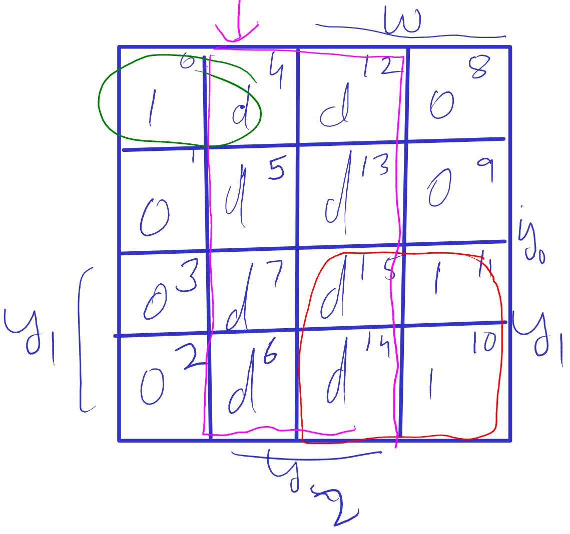
where $y_2=1$ put $K_2=\overline{Y_2^+}$

Y_2^+



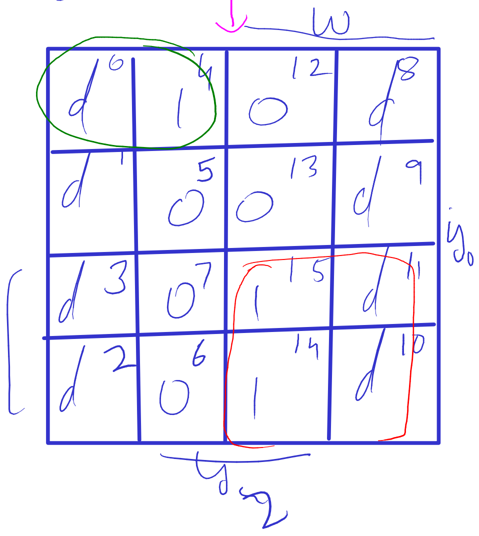
1^{0}	0^{4}	1^{12}	0^{8}
0^{1}	1^{5}	1^{13}	0^{9}
0^{3}	1^{7}	0^{15}	1^{11}
0^{2}	1^{6}	0^{14}	1^{10}

J_2



1^{0}	d^{4}	d^{12}	0^{8}
0^{1}	d^{5}	d^{13}	0^{9}
0^{3}	d^{7}	d^{15}	1^{11}
0^{2}	d^{6}	d^{14}	1^{10}

K_2



d^{0}	1^{4}	0^{12}	d^{8}
d^{1}	0^{5}	0^{13}	d^{9}
d^{3}	0^{7}	1^{15}	d^{11}
d^{2}	0^{6}	1^{14}	d^{10}

y_2	Y_2	J_2	K_2
0	0	0	d
0	1	1	d
1	0	d	1
1	1	d	0

$J_2=\bar{w}\bar{y}_1\bar{y}_0+wy_1$

$K_2=\bar{w}\bar{y}_1\bar{y}_0+wy_1$

Y_1^+

w

1^{0}	1^{4}	1^{12}	1^{8}
0^{1}	0^{5}	1^{13}	1^{9}
1^{3}	1^{7}	0^{15}	0^{11}
0^{2}	0^{6}	0^{14}	0^{10}

y_0

y_1

y_2

J_1

w

1^{0}	1^{4}	1^{12}	1^{8}
0^{1}	0^{5}	1^{13}	1^{9}
d^{3}	d^{7}	d^{15}	d^{11}
d^{2}	d^{6}	d^{14}	d^{10}

y_0

y_1

y_2

$\overline{J_1} = \overline{w}\, y_0$

$\rightarrow J_1 = w + \overline{y}_0$

K_1

w

d^{0}	d^{4}	d^{12}	d^{8}
d^{1}	d^{5}	d^{13}	d^{9}
0^{3}	0^{7}	1^{15}	1^{11}
1^{2}	1^{6}	1^{14}	1^{10}

y_1

y_2

$\overline{K_1} = \overline{w}\, y_0$

$\rightarrow K_1 = w + \overline{y}_0$

Y_0^+

w

1^{0}	1^{4}	0^{12}	0^{8}
0^{1}	0^{5}	1^{13}	1^{9}
0^{3}	0^{7}	1^{15}	1^{11}
1^{2}	1^{6}	0^{14}	0^{10}

y_0

y_1

y_2

J_0

w

1^{0}	1^{4}	0^{12}	0^{8}
d^{1}	d^{5}	d^{13}	d^{9}
d^{3}	d^{7}	d^{15}	d^{11}
1^{2}	1^{6}	0^{14}	0^{10}

y_0

y_1

y_2

$J_0 = \overline{w}$

K_0

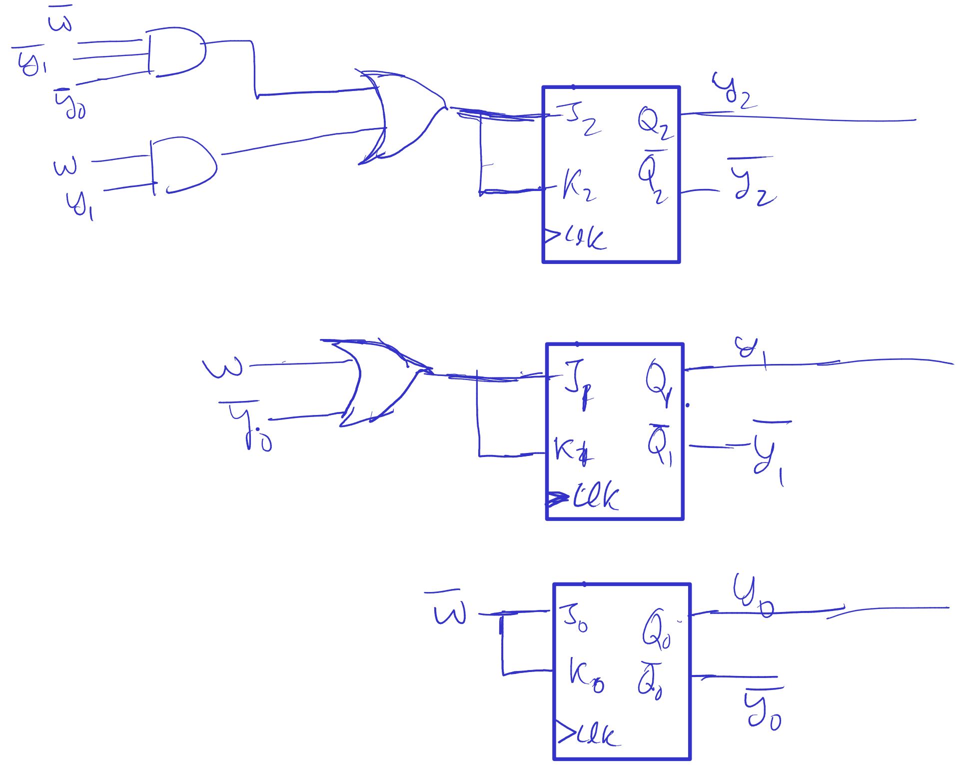
w

d^{0}	d^{4}	d^{12}	d^{8}
1^{1}	1^{5}	0^{13}	0^{9}
1^{3}	1^{7}	0^{15}	0^{11}
d^{2}	d^{6}	d^{14}	d^{10}

y_1

y_2

$K_0 = \overline{w}$



w̄
ȳ1
ȳ0
w
y1
J2
K2
clk
Q2
Q̄2
y2
ȳ2
w
ȳ0
J1
K1
clk
Q1
Q̄1
y1
ȳ1
w̄
J0
K0
clk
Q0
Q̄0
y0
ȳ0

Problem 4

① Row reduction
② Implication table/chart

Two states are equivalent if
next states for every input and outputs
for every input are equivalent

Implication table/chart

a								
b	e≡c							
c	X	X						
d	~~e≡h e≡a~~	~~c≡h e≡a~~	X					
e	X	X	h≡f	X				
f	X	X	~~i≡e h≡g~~	X	~~i≡e f≡g~~			
g	~~e≡h c≡b~~	~~c≡h e≡b~~	X	a≡b	X	X		
h	X	X	~~i≡c h≡d~~	X	~~i≡c f≡d~~	c≡c g≡d	X	
i	~~e≡f e≡b~~	~~c≡f e≡b~~	X	h≡f a≡b	X	X	h≡f	X
	a	b	c	d	e	f	g	h	i

Present State	Next State $X = 0$	1	Present Output (Z)
a	e	e	1
b	c	e	1
c	i	h	0
d	h	a	1
e	i	f	0
f	e	g	0
g	h	b	1
h	c	d	0
i	f	b	1

① X out different states due to output being different

② Fill out the cells with conditions that need to be true for the corresponding states to be equivalent.

③ cross out impossible conditions

Equivalance $\underbrace{a \equiv b}_{a}$, $\underbrace{c \equiv e}_{c}$, $\underbrace{d \equiv i \equiv g}_{d}$, $\underbrace{f \equiv h}_{f}$

Reduced state table

PS	NS $x=0$	$x=1$	Output (z)
a	c	c	1
c	d	f	0
d	f	g	1
f	c	d	0

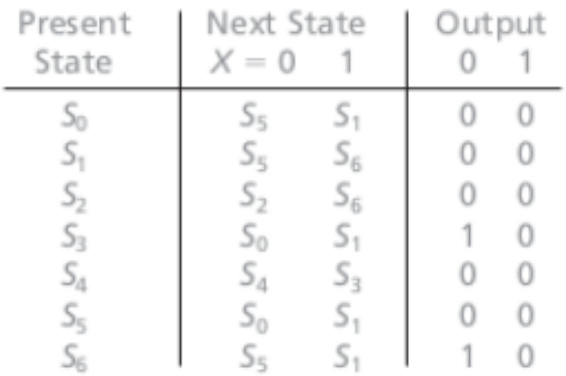
Problem 5.1

X = Due to mismatching output

X = Due to $S_3 \neq$ *

X = Due to $S_6 \neq$ *

	S_0	S_1	S_2	S_3	S_4	S_5	S_6
S_0							
S_1	~~$S_1 \equiv S_6$~~						
S_2	~~$S_2 \equiv S_5$~~	~~$S_2 \equiv S_5$~~					
S_3	X	X	X				
S_4	~~$S_4 \equiv S_5$~~ ~~$S_3 \equiv S_1$~~	~~$S_4 \equiv S_5$~~ ~~$S_3 \equiv S_6$~~	$S_3 \equiv S_6$ ✓	X			
S_5	✓	~~$S_0 \equiv S_5$~~ ~~$S_1 \equiv S_6$~~	~~$S_0 \equiv S_2$~~ ~~$S_1 \equiv S_6$~~	X	~~$S_0 \equiv S_4$~~ ~~$S_1 \equiv S_3$~~		
S_6	X	X	X	$S_0 \equiv S_5$ ✓	X	X	



Present State	Next State $X = 0$	1	Output 0	1
S_0	S_5	S_1	0	0
S_1	S_5	S_6	0	0
S_2	S_2	S_6	0	0
S_3	S_0	S_1	1	0
S_4	S_4	S_3	0	0
S_5	S_0	S_1	0	0
S_6	S_5	S_1	1	0

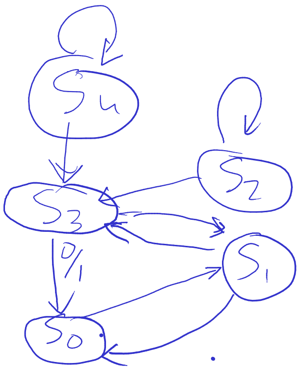
Replace S_5 with S_0, S_6 with S_3. State reduction of BINary state table still leaves us with 5 states instead of 7.

FL Ipflop's state table has only 3 states.

Mr Ipflop is not correct.

PS	NS $X=0$	1	Output $X=0$	$X=1$
S_0	S_0	S_1	0	0
S_1	S_0	S_3	0	0
S_2	S_2	S_3	0	0
S_3	S_0	S_1	1	0
S_4	S_4	S_3	0	0

Problem 5.2

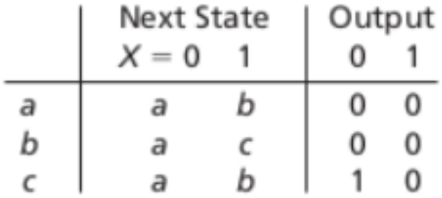


If S_0 is the start state S_2, S_4 are unreachable.

Remove S_2, S_4 from the state table

Combining with $S_5 \equiv S_0$, $S_3 \equiv S_6$ we get:

PS	NS X=0	1	Output X=0	1
S_0	S_0	S_1	0	0
S_1	S_0	S_3	0	0
S_3	S_0	S_1	1	0



	Next State X = 0	1	Output 0	1
a	*a*	*b*	0	0
b	*a*	*c*	0	0
c	*a*	*b*	1	0

Comparing with Ipflop's table

$S_3 \equiv c$ because output (1,0) is unique

$S_1 \equiv b$

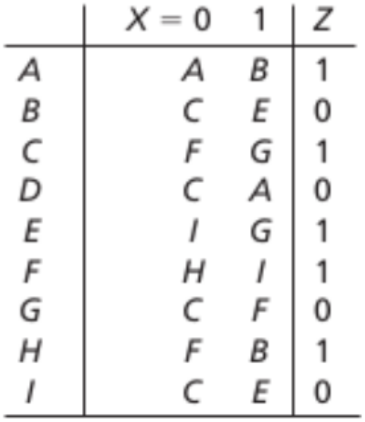
$S_0 \equiv a$

In this case Mr Ip flop is correct

Problem 6

✗ = Due to mismatch in output

	A	B	C	D	E	F	G	H
B	✗							
C	~~A≡F B≡G~~	✗						
D	✗	~~E≡A~~	✗					
E	~~A≡I B≡G~~	✗	~~F≡I~~	✗				
F	A≡H B≡I ✓	✗	~~F≡H G≡I~~	✗	~~I≡H G≡I~~			
G	✗	~~E≡F~~	✗	A≡F ✓	✗	✗		
H	A≡F ✓	✗	~~G≡B~~	✗	~~I≡F G≡B~~	I≡B ✓	✗	
I	✗	✓	✗	~~A≡E~~	✗	✗	~~E≡F~~	✗



	X = 0	1	Z
A	A	B	1
B	C	E	0
C	F	G	1
D	C	A	0
E	I	G	1
F	H	I	1
G	C	F	0
H	F	B	1
I	C	E	0

A≡H≡F B≡I D≡G

Replace H and F with A

I with B

G with D

PS	NS X=0	1	Z
A	A	B	1
B	C	E	0
C	A	D	1
D	C	A	0
E	B	D	1

Problem 6.2

Highest Priority: (A, C) ✓, (B, D) ✓, (C, E) ✓,

A C D

Medium Priority: (A, B) ✓, (C, E) ✓, (A, D), (C, A) ✓

(B, D) ✓

State Map $y_2 y_1$ / y_0

y_2

A 0	2	6	C 4
B 1	D 3	7	E 5

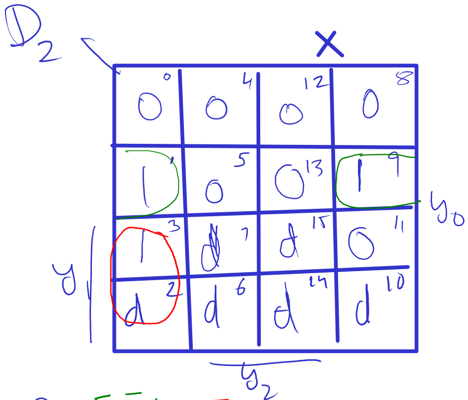
y_0 (second row), y_1 (middle two columns)

	X = 0	1	Z
A	A	B	1
B	C	E	0
C	F	G	1
D	C	A	0
E	I	G	1
F	H	I	1
G	C	F	0
H	F	B	1
I	C	E	0

Assignment

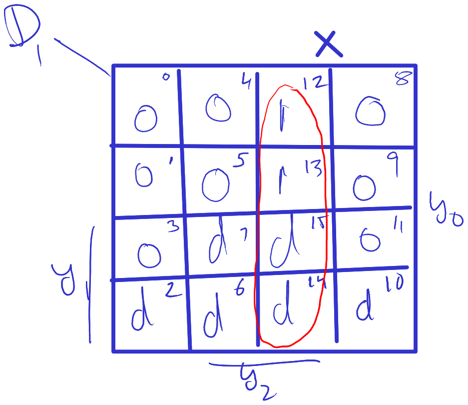
y_2	y_1	y_0	State
0	0	0	A
0	0	1	B
~~0~~	~~1~~	~~0~~	
0	1	1	D
1	0	0	C
1	0	1	E
~~1~~	~~1~~	~~0~~	
~~1~~	~~1~~	~~1~~	

	PS			NS						Output (Z)
				X = 0			X = 1			
	y_2	y_1	y_0	Y_2	Y_1	Y_0	Y_2	Y_1	Y_0	
A=H=F	0	0	0	0	0	0	0	0	1	1
B=I	0	0	1	1	0	0	1	0	1	0
	0	1	0	d	d	d	d	d	d	d
D=G	0	1	1	1	0	0	0	0	0	0
C	1	0	0	0	0	0	0	1	1	1
E	1	0	1	0	0	1	0	1	1	1
	1	1	0	d	d	d	d	d	d	d
	1	1	1	d	d	d	d	d	d	d



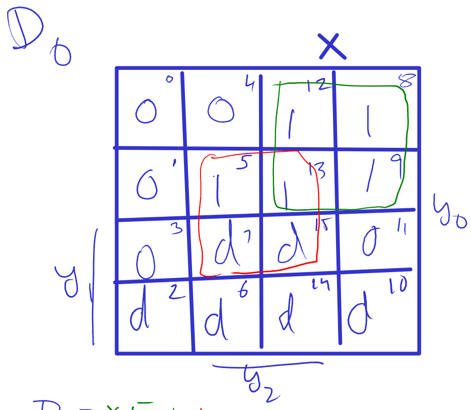
0	0	0	0
1	0	0	1
1	d	d	0
d	d	d	d

$D_2 = \bar{y}_2\bar{y}_1y_0 + \bar{x}\bar{y}_2y_1$



0	0	1	0
0	0	1	0
0	d	d	0
d	d	d	d

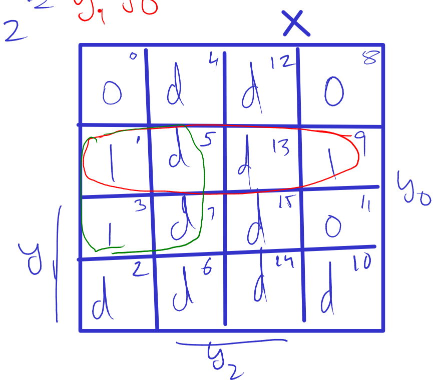
$D_1 = xy_2$



0	0	1	1
0	1	1	1
0	d	d	0
d	d	d	d

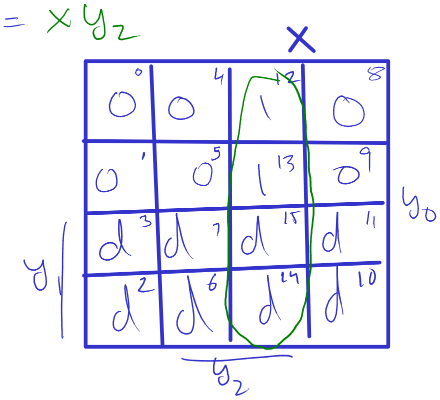
$D_0 = x\bar{y}_1 + y_2y_0$

$J_2 = \bar{y}_1y_0 + \bar{x}y_0$



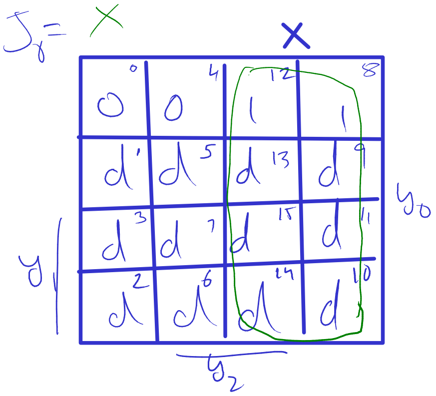
0	d	d	0
1	d	d	1
1	d	d	0
d	d	d	d

$J_1 = xy_2$



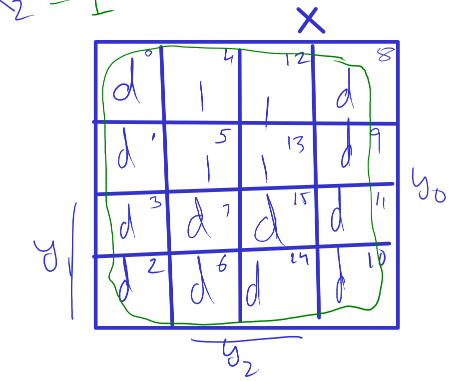
0	0	1	0
0	0	1	0
d	d	d	d
d	d	d	d

$J_0 = x$



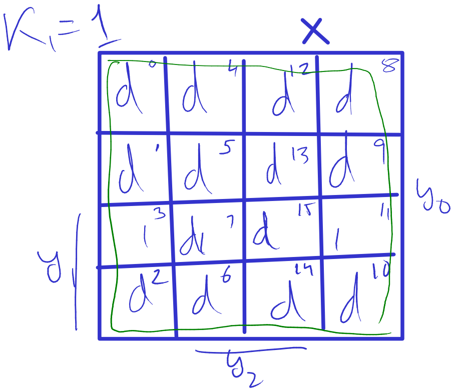
0	0	1	1
d	d	d	d
d	d	d	d
d	d	d	d

$K_2 = 1$



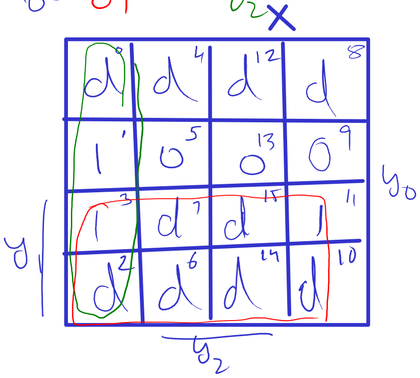
d	1	1	d
d	1	1	d
d	d	d	d
d	d	d	d

$K_1 = 1$



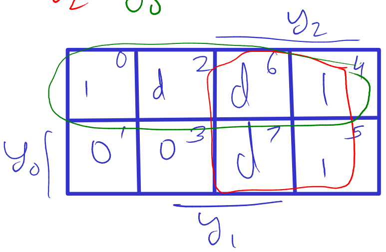
d	d	d	d
d	d	d	d
1	d	d	1
d	d	d	d

$K_0 = y_1 + \bar{x}y_2$



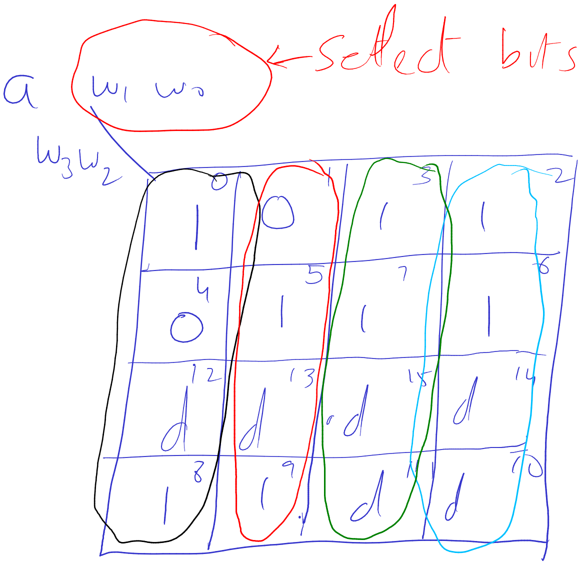
d	d	d	d
1	0	0	0
1	d	d	1
d	d	d	d

$Z = y_2 + \bar{y}_0$

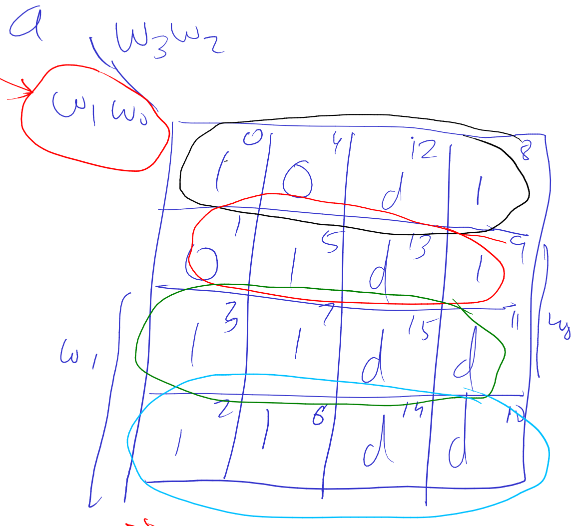


1	d	d	1
0	0	d	1

Prob 7
Part 2

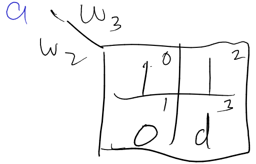


← select bits

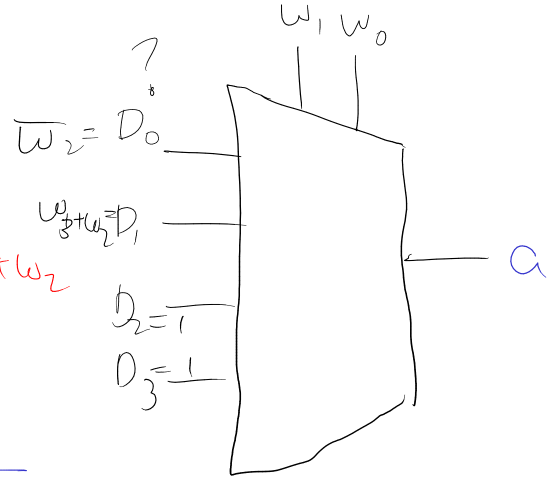


← two approaches

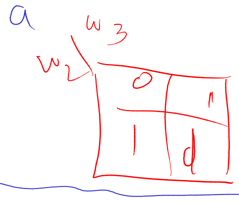
when $w_1 w_0 = 0\,0$



$a = D_0 = \overline{w_2}$



when $w_1 w_0 = 01$

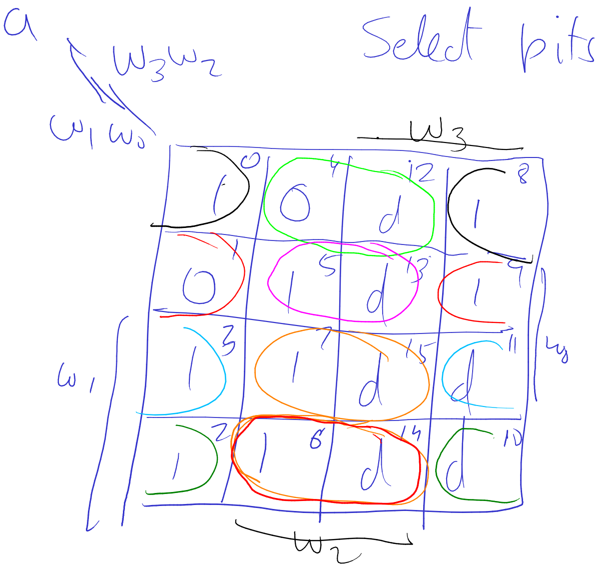


$a = D_1 = w_3 + w_2$

when $w_1 w_0 = 10$ $a = D_2 = 1$

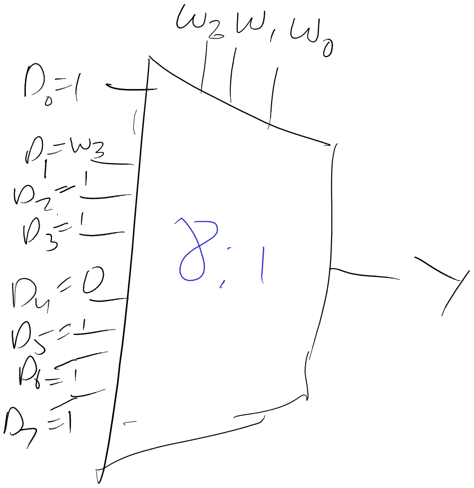
when $w_1 w_0 = 11$ $a = D_3 = 1$

Prob 7
Part 1

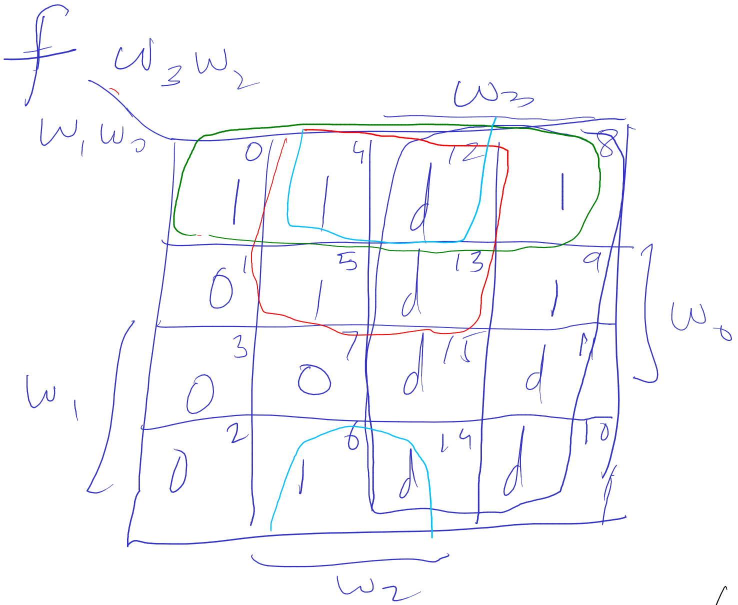


Select bits $w_2 w_1 w_0$

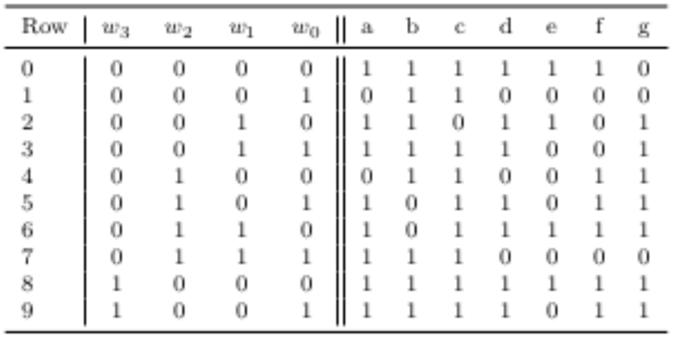
when w_2	w_1	w_0	Y
0	0	0	$D_0 = 1$
0	0	1	$D_1 = w_3$
0	1	0	$D_2 = 1$
0	1	1	$D_3 = 1$
1	0	0	$D_4 = 0$
1	0	1	$D_5 = 1$
1	1	0	$D_6 = 1$
1	1	1	$D_7 = 1$



Problem 7, Part 3



0 1	4 1	12 d	8 1
1 0	5 1	13 d	9 1
3 0	7 0	15 d	11 d
2 0	6 1	14 d	10 d

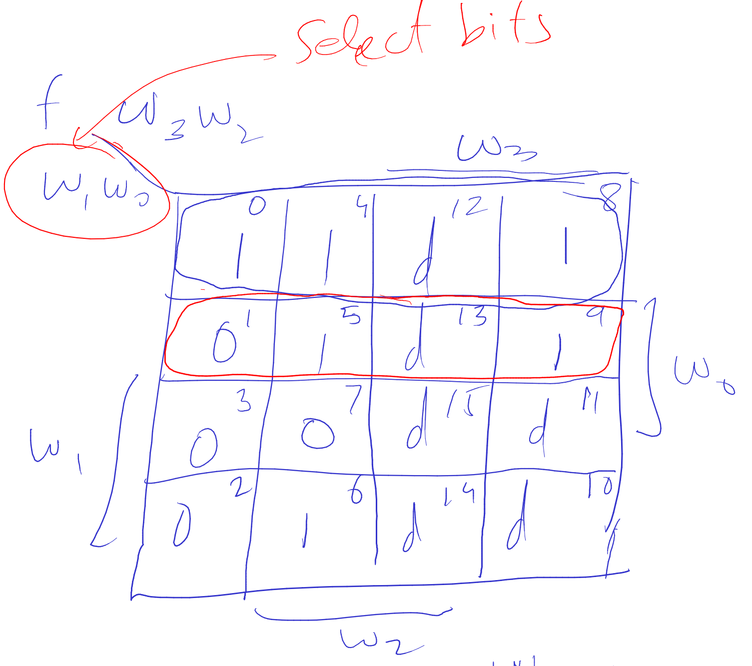


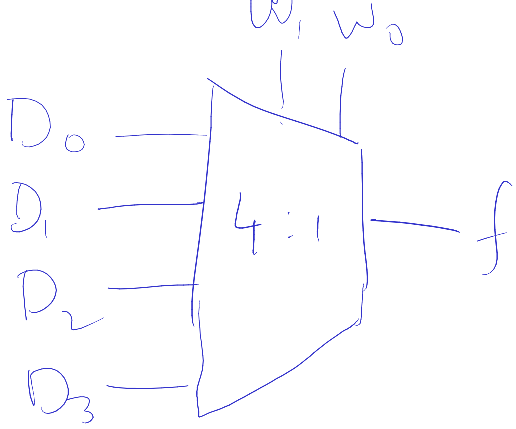
Row	w_3	w_2	w_1	w_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1

$$f = w_3 + \overline{w_1}\, w_2 + \overline{w_0}\, \overline{w_1} + \overline{w_0}\, w_2$$

w_3 occurs 1 times in f
w_2 " 2 " " f
w_1 " 2 " " f
w_0 " 2 " " f

Pick $w_1 w_0$
as select bits



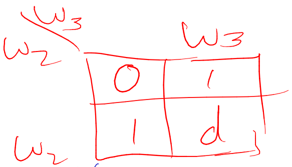


when $w_1 w_0 = 00$

$f = D_0 = 1$

when $w_1 w_0 = 01$

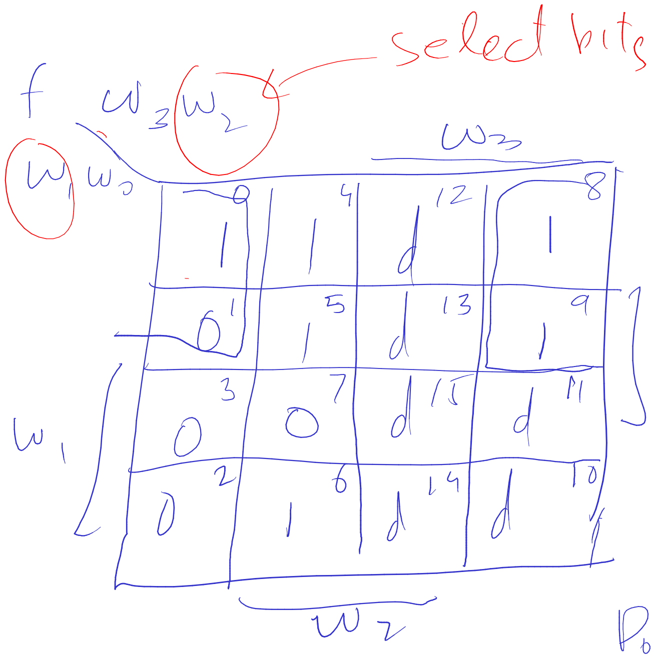
$f = D_1 = ?$

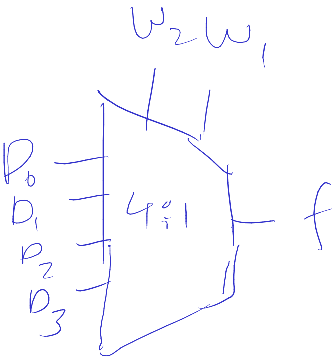


$f = D_1 = w_2 + w_3$

FAIL: Problem says cannot use another gate

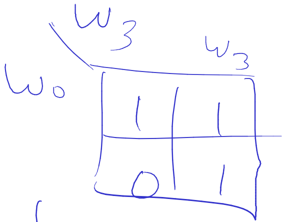
Pick $w_2 w_1$ as the select bits





when $w_2 w_1 = 00$

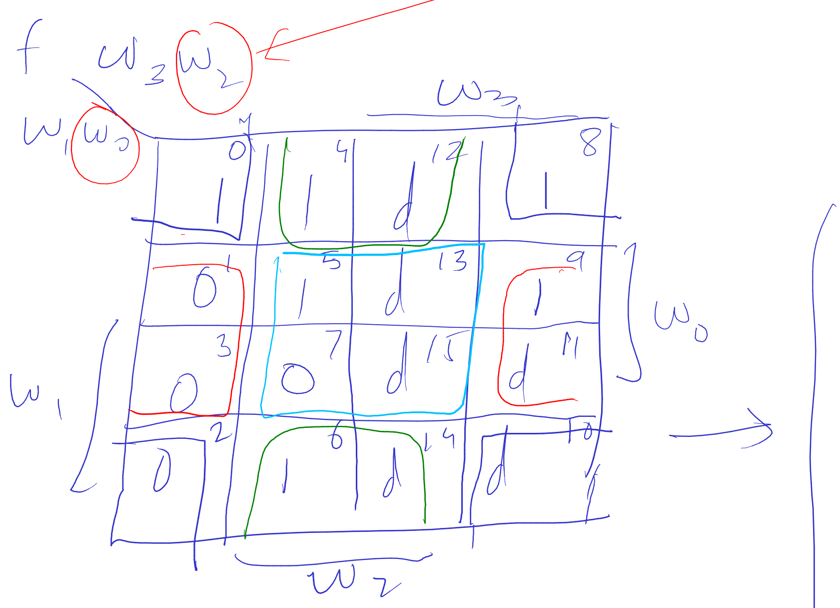
$f = D_0 = ?$



$f = D_0 = \overline{w_0} + w_3$

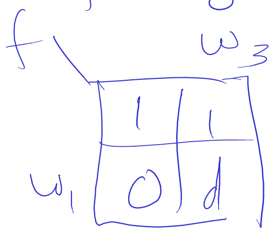
FAIL problem says cannot use another gate

Pick $w_2 w_0$ as the select bits



when $w_2 w_0 = 00$

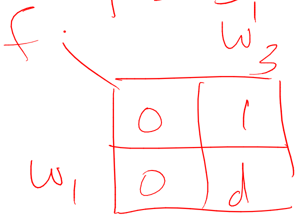
$f = D_0 = ?$



$f = D_0 = \overline{w_1}$

when $w_2 w_0 = 01$

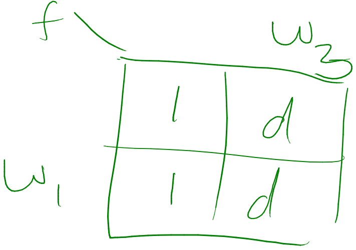
$f = D_1 = ?$



$f = D_1 = w_3$

when $w_2 w_0 = 10$

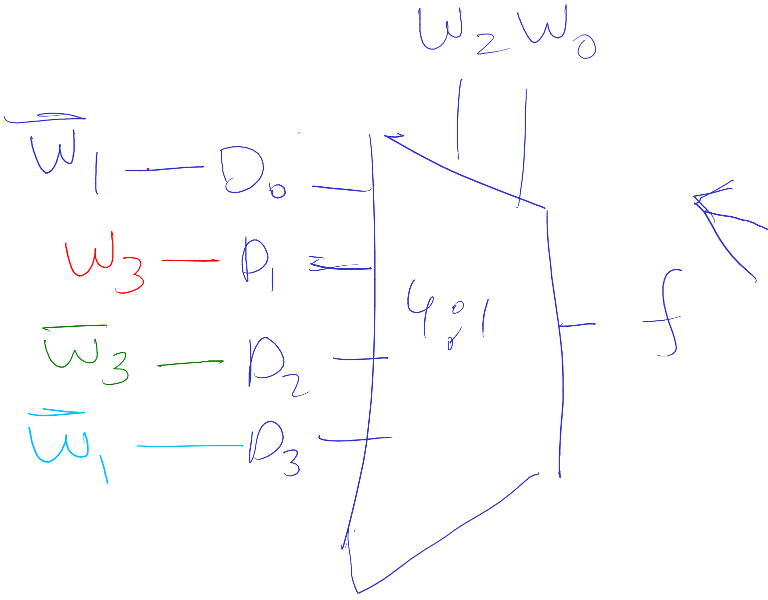
$f = D_2 = ?$



$f = D_2 = \overline{w_3}$

when $w_2 w_0 = 11$

$f = D_3 = ? = \overline{w_1}$



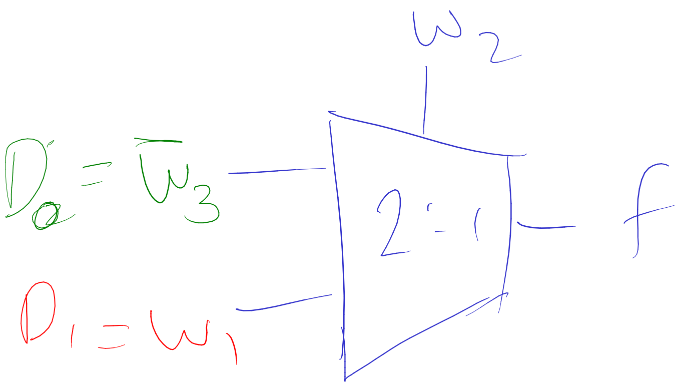
SUCCESS No other gates needed

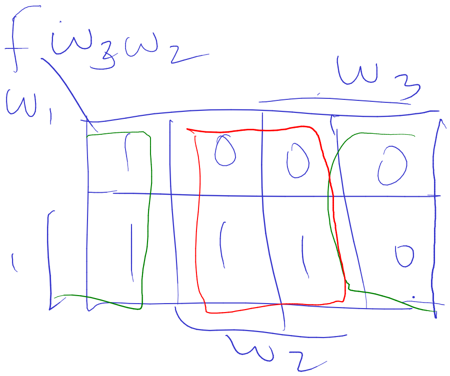
Problem 8

$f = \overline{w_2}\,\overline{w_3} + w_1 w_2$

w_2 occurs 2 times
w_3 " 1 "
w_1 " 1 "

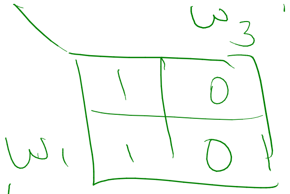
Pick w_2 as the select bit





when $w_2 = 0$

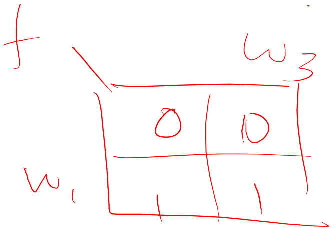
$f = D_0 = ?$



$f = \overline{w_3} = D_0$

when $w_2 = 1$

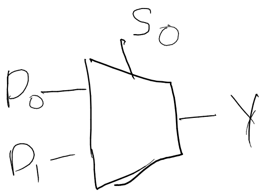
$f = D_1 = ?$



$f = w_1 = D_1$

Method 2: 2:1 MUX equation is

$Y = \overline{S_0} D_0 + S_0 D_1$



compare with $f = \overline{w_2}\,\overline{w_3} + w_1 w_2$

$$f = \overline{w_2}\,\overline{w_3} + w_2 w_1$$

$$Y = \overline{S_0} D_0 + S_0 D_1$$

<u>Problem 9</u> $f(w_1, w_2, w_3) = \sum m(0, 2, 3, 4, 5, 7)$