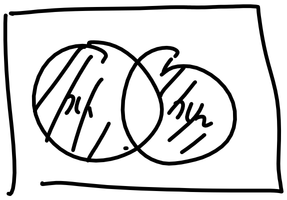
8 Karnaugh maps = (truth table + Venn diagram)

8.1 Two input K-maps

B \ A	0	1
0	m_0	m_2
1	m_1	m_3

XOR gate

x_1	x_2	f
0	0	0
0	1	1
1	0	1
1	1	0



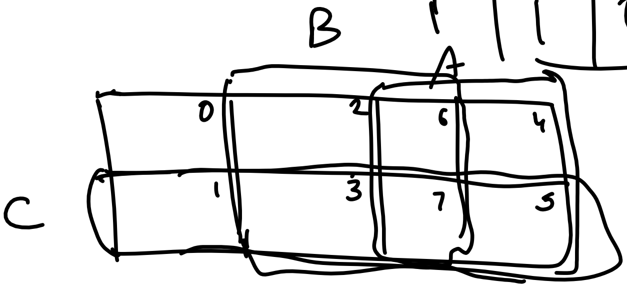
	$\bar{x}_2$	x_2
$\bar{x}_1$	0	1
x_1	1	0

x_2 \ x_1	0	1
0	0	1
1	1	0

8.2 Three input K-maps

$0 \quad 1 \quad 3 \rightarrow 2$

C \ AB	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5



8.3 Four input K-maps

CD \ AB	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

8.4 Five input K-maps

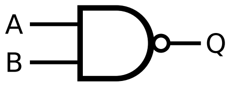
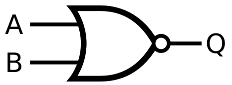
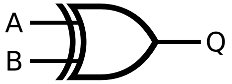
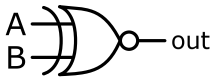
A = 0

DE \ BC	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

A = 1

DE \ BC	00	01	11	10
00	m_{16}	m_{20}	m_{28}	m_{24}
01	m_{17}	m_{21}	m_{29}	m_{25}
11	m_{19}	m_{23}	m_{31}	m_{27}
10	m_{18}	m_{22}	m_{30}	m_{26}

9 More Gates and notations summary

Name	C/Verilog	Boolean expr.	Truth Table	(ANSI) symbol	K-map
NAND Gate	`Q = ~(x1 & x2)`	$Q = \overline{x_1 \cdot x_2} = \overline{x_1 x_2}$	x_1 x_2 | $\overline{x_1 \cdot x_2}$ 0 0 | 1 0 1 | 1 1 0 | 1 1 1 | 0		B\A: 0 1 0: 1 1 1: 1 0
NOR Gate	`Q = ~(x1 | x2)`	$Q = \overline{x_1 + x_2}$	x_1 x_2 | $\overline{x_1 + x_2}$ 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 0		B\A: 0 1 0: ~~0~~ 1 0 1: 0 ~~1~~ 0
XOR Gate	`Q = x1 ^ x2`	$Q = x_1 \oplus x_2$	x_1 x_2 | $x_1 \oplus x_2$ 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 0		B\A: 0 1 0: 0 1 1: 1 0
XNOR Gate	`Q = ~(x1 ^ x2)`	$Q = \overline{x_1 \oplus x_2}$	x_1 x_2 | $\overline{x_1 \oplus x_2}$ 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 1		B\A: 0 1 0: 1 0 1: 0 1

NAND gate

A	B	f
0	0	1
0	1	1
1	0	1
1	1	0

K-map

B \ A	$\bar{A}$ $A=0$	A $A=1$
$\bar{B}$ $B=0$	1	1
B $B=1$	1	0

NOR gate

OR → NOT

$g = A+B$

A	B	g	f
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

K-map

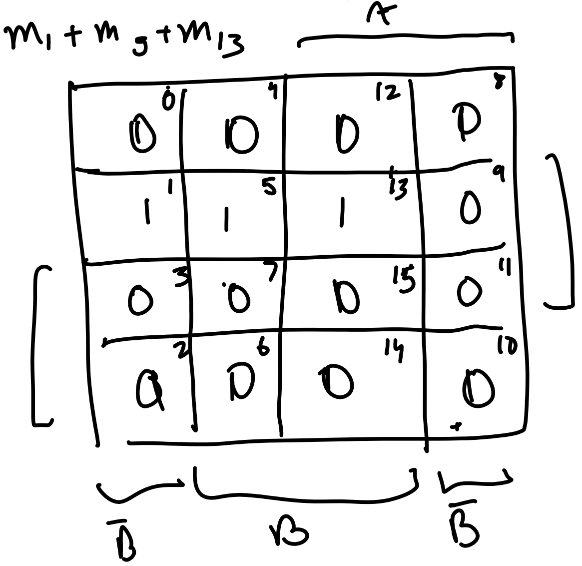
B ↓ \ A →	0	1
0	1	0
1	0	0

$A, \overrightarrow{B, C}, D$

Example 10. *Convert the following Boolean expression into a K-map.* $f = \overline{A\bar{B} + CD}$

$f = m_1 + m_5 + m_{13}$

$f =$



0	0	0	0
1	1	1	0
0	0	0	0
0	0	0	0

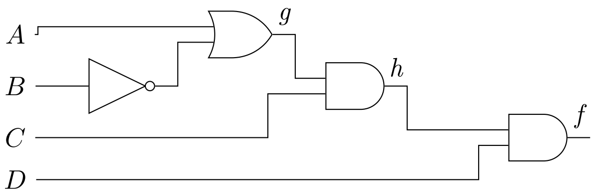
$g_1 = A\bar{B}$

$g_2 = g_1 + C$

$g_3 = \overline{g_2}$

$f = g_3 \cdot D$

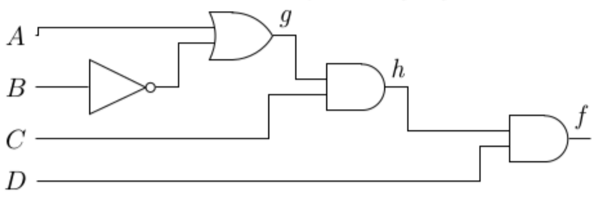
Problem 10. *Convert the following logic circuit into a K-map.*



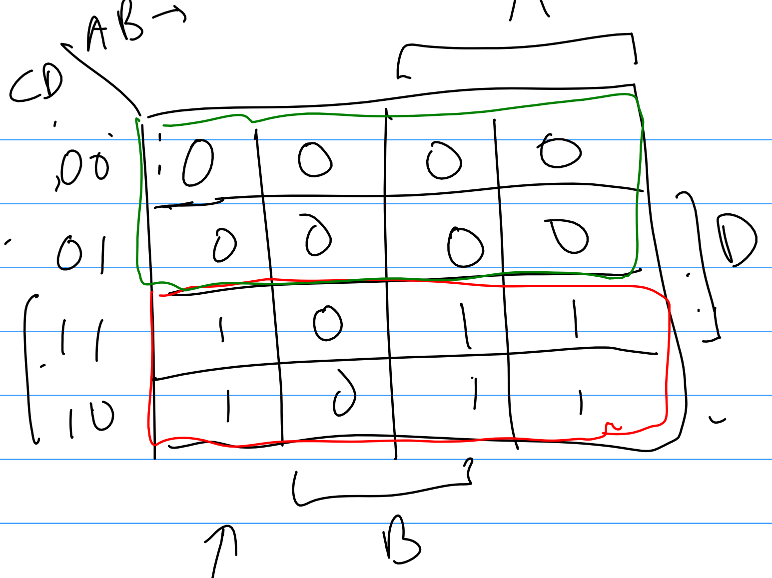
10 Boolean Algebra

10.1 Axioms of Boolean algebra

1. $0 \cdot 0 = 0$
2. $1 + 1 = 1$



ABCD



AB→ / CD	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	1	0	1	1
10	1	0	1	1

A

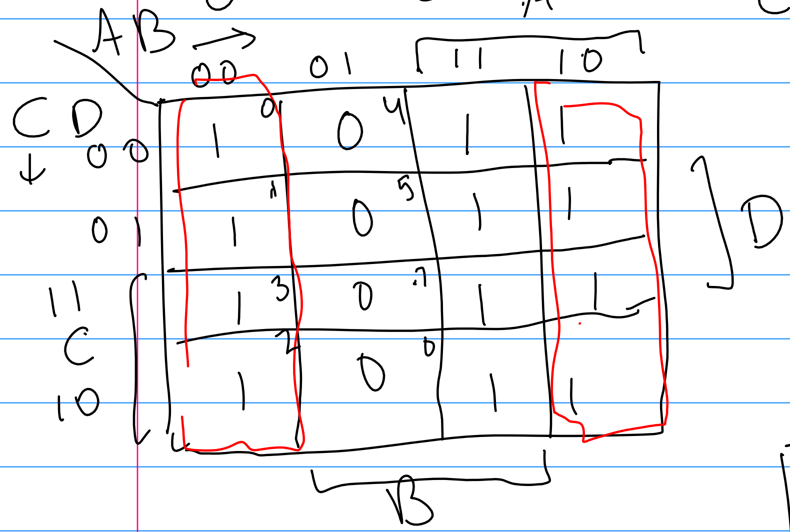
B

C

D

g=

$g = A + \overline{B}$



AB→ / CD ↓	00	01	11	10
00	1^{0}	0^{4}	1	1
01	1^{1}	0^{5}	1	1
11	1^{3}	0^{7}	1	1
10	1^{2}	0^{6}	1	1

A

B

C

D

$h = g \cdot C$

$$\begin{bmatrix}1\\1\\1\\1\end{bmatrix} \cdot \begin{bmatrix}0\\0\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\\1\\1\end{bmatrix}$$

A =

AB

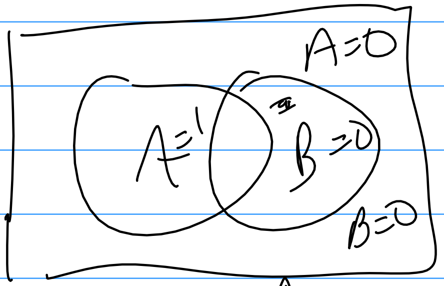
0	0	1	1
0	0	1	1
0	0	1	1
0	0	1	1

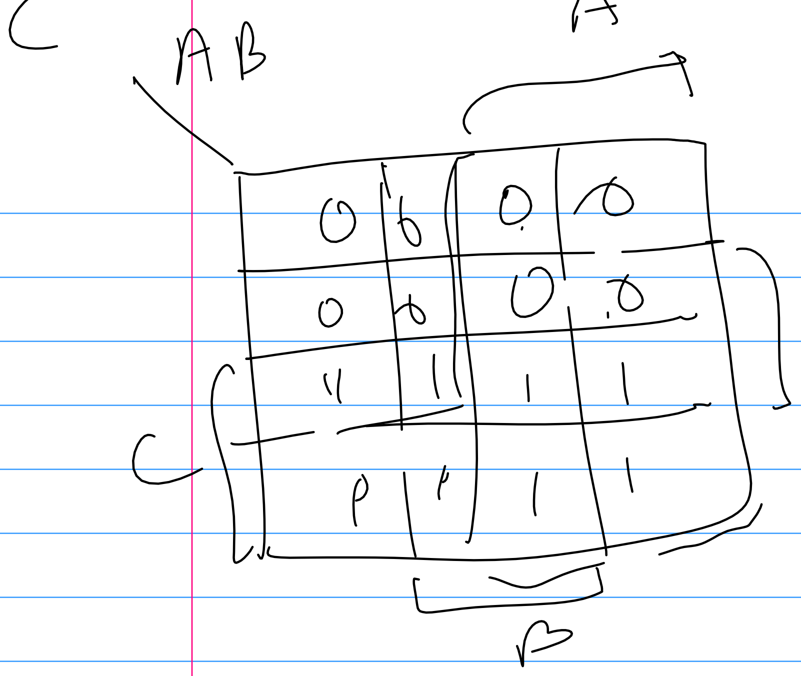
A

B

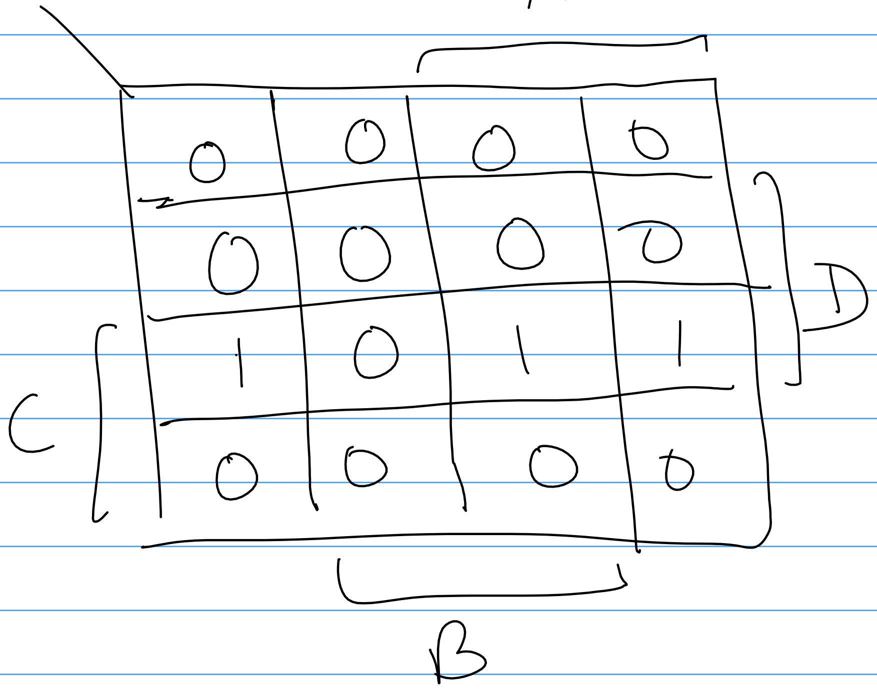
C

D





f =



$f = h \cdot D$

$= (A + \overline{B}) C \cdot D$

3. $1 \cdot 1 = 1$
4. $0 + 0 = 0$
5. $0 \cdot 1 = 1 \cdot 0 = 0$
6. $\bar{0} = 1$
7. $\bar{1} = 0$
8. $x = 0$ if $x \neq 1$
9. $x = 1$ if $x \neq 0$

10.2 Single variable theorems (Prove by drawing K-maps)

1. $x \cdot 0 = 0$

2. $x + 1 = 1$

3. $x \cdot 1 = x$

4. $x + 0 = x$

5. $x \cdot x = x$

6. $x + x = x$

7. $x \cdot \bar{x} = 0$

8. $x + \bar{x} = 1$

9. $\bar{\bar{x}} = x$

Remark 2 (Duality)**.** *Swap* $+$ *with* $\cdot$ *and 0 with 1 to get another theorem*

10.3 Two and three variable properties (Prove by K-maps)

1. Commutative: $x \cdot y = y \cdot x$, $x + y = y + x$

2. Associative: $x \cdot (y \cdot z) = (x \cdot y) \cdot z$, $x + (y + z) = (x + y) + z$

3. Distributive: $x \cdot (y + z) = x \cdot y + x \cdot z$, $x + y \cdot z = (x + y) \cdot (y + z)$

4. Absorption: $x + x \cdot y = x$, $x \cdot (x + y) = x$

5. Combining: $x \cdot y + x \cdot \bar{y}$, $(x + y) \cdot (x + \bar{y}) = x$

6. DeMorgan's theorem: $\overline{x \cdot y} = \bar{x} + \bar{y}$, $\overline{x + y} = \bar{x} \cdot \bar{y}$.

7. Concensus:

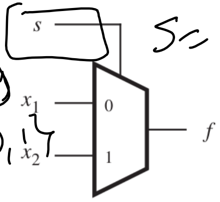
   (a) $x + \bar{x} \cdot y = x + y$

   (b) $x \cdot (\bar{x} + y) = x \cdot y$

   (c) $x \cdot y + y \cdot z + \bar{x} \cdot z = x \cdot y + \bar{x} \cdot z$

   (d) $(x + y) \cdot (y + z) \cdot (\bar{x} + z) = (x + y) \cdot (\bar{x} + z)$

Example 11 (Multiplexer)**.** *Multiplexer is a circuit used to select one of the input lines x_1 and x_2 based only select input s. When $s = 0$, x_1 is selected, x_2 is selected otherwise. Find a boolean expression and a circuit for multiplexer*



$s = 0$

$x_1 \in \{0,1\}$

$x_2 \in \{0,1\}$

s	$f(s, x_1, x_2)$
0	x_1
1	x_2

s	x_1	x_2	f
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Example 12. *Simplify* $f = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}\bar{C}$ *using boolean algebra*

Example 13. *Simplify* $f = \bar{A}\bar{A}\bar{C} + \bar{A}\bar{B}C$ *using K-maps.*

s, x1 x2 →

↓	s	x_1	x_2	f
0	0	0	0	0
1	0	0	1	0
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	0
7	1	1	1	1

$s x_1 \rightarrow$ s

$x_2 \downarrow$

	^{0} 0	^{2} 1	^{6} 0	^{4} 0
x_2	^{1} 0	^{3} 1	^{7} 1	^{5} 1

x_1

$s x_1 \rightarrow$ 00 01 10 11

$x_2 \downarrow$

$x_1 x_2 \rightarrow$

$s \downarrow$

	^{0}	^{1}		^{2}
s				

x_2

x_1	x_2	s	f

---

f

$s x_1 \rightarrow$ $\bar{s}$ $\bar{s}$ s

$x_2 \downarrow$

	^{0} 0	^{2} 1	^{6} 0	^{4} 0
x_2	^{1} 0	^{3} 1	^{7} 1	^{5} 1

x_1

✓ $f = m_2 + m_3 + m_5 + m_7$

✓ $f = x_1 \cdot \bar{s}$

$+ x_2 x_1$ ← unnecessary

$+ x_2 \cdot s$

$f = x_1 \cdot \bar{s} + x_2 s$