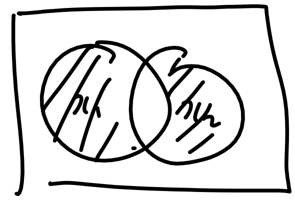
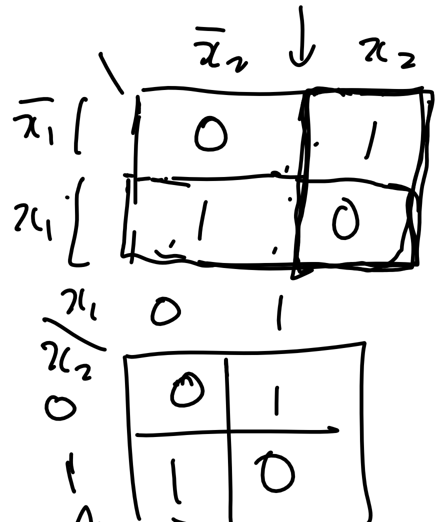
8 Karnaugh maps = (truth table + Venn diagram)

XOR gate

x_1	x_2	f
0	0	0
0	1	1
1	0	1
1	1	0





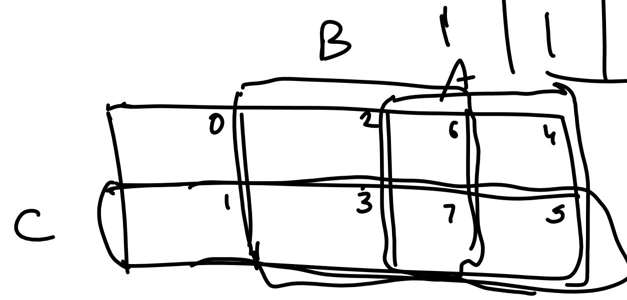
8.1 Two input K-maps

B \ A	0	1
0	m_0	m_2
1	m_1	m_3

8.2 Three input K-maps

0 1 3 → 2

C \ AB	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5



8.3 Four input K-maps

CD \ AB	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

8.4 Five input K-maps

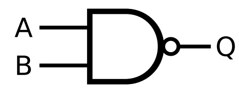
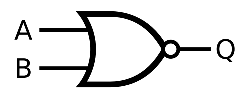
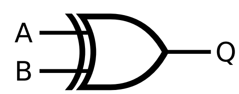
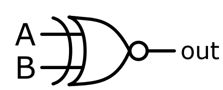
A = 0

DE \ BC	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

A = 1

DE \ BC	00	01	11	10
00	m_{16}	m_{20}	m_{28}	m_{24}
01	m_{17}	m_{21}	m_{29}	m_{25}
11	m_{19}	m_{23}	m_{31}	m_{27}
10	m_{18}	m_{22}	m_{30}	m_{26}

9 More Gates and notations summary

Name	C/Verilog	Boolean expr.	Truth Table	(ANSI) symbol	K-map
NAND Gate	`Q = ~(x1 & x2)`	$Q = \overline{x_1 \cdot x_2} = \overline{x_1 x_2}$	x_1 x_2 | $\overline{x_1 \cdot x_2}$ 0 0 | 1 0 1 | 1 1 0 | 1 1 1 | 0		A: 0 1 B=0: 1 1 B=1: 1 0
NOR Gate	`Q = ~(x1 | x2)`	$Q = \overline{x_1 + x_2}$	x_1 x_2 | $\overline{x_1 + x_2}$ 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 0		A: 0 1 B=0: 1 0 B=1: 0 1
XOR Gate	`Q = x1 ^ x2`	$Q = x_1 \oplus x_2$	x_1 x_2 | $x_1 \oplus x_2$ 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 0		A: 0 1 B=0: 0 1 B=1: 1 0
XNOR Gate	`Q = ~(x1 ^ x2)`	$Q = \overline{x_1 \oplus x_2}$	x_1 x_2 | $\overline{x_1 \oplus x_2}$ 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 1		A: 0 1 B=0: 1 0 B=1: 0 1

NAND gate

A	B	f
0	0	1
0	1	1
1	0	1
1	1	0

K-map

B \ A	$\bar{A}$ (A = 0)	A (A = 1)
$\bar{B}$ (B = 0)	1	1
B (B = 1)	1	0

NOR gate

OR → NOT

$g = A + B$

A	B	g	f
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

K-map

B ↓ \ A →	0	1
0	1	0
1	0	0

$\overrightarrow{A, B, C, D}$

Example 10. *Convert the following Boolean expression into a K-map.* $f = \overline{A\bar{B} + CD}$

$f = m_1 + m_5 + m_{13}$

$f =$

	$\bar{B}$	B	B, A	$\bar{B}$, A
	0 (0)	0 (4)	0 (12)	0 (8)
D	1 (1)	1 (5)	1 (13)	0 (9)
C, D	0 (3)	0 (7)	0 (15)	0 (11)
C	0 (2)	0 (6)	0 (14)	0 (10)

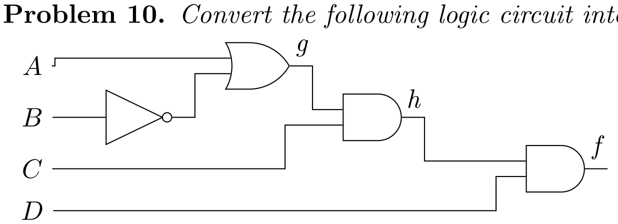
$g_1 = A\bar{B}$

$g_2 = g_1 + C$

$g_3 = \overline{g_2}$

$f = g_3 \cdot D$

Problem 10. *Convert the following logic circuit into a K-map.*



10 Boolean Algebra

10.1 Axioms of Boolean algebra

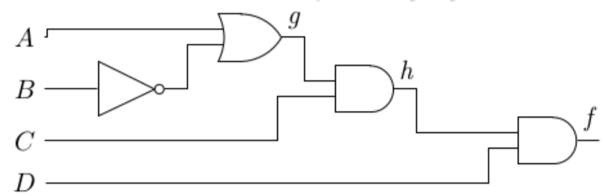
1. $0 \cdot 0 = 0$
2. $1 + 1 = 1$

$0 \cdot 0 = 0$

$0 \cdot 1 = 0$

$1 \cdot 0 = 0$

$1 \cdot 1 = 1$



$g =$

$g = A + \overline{B}$

CD \ AB	00	01	11	10
00	1	0	1	1
01	1	0	1	1
11	1	0	1	1
10	1	0	1	1

A B C D

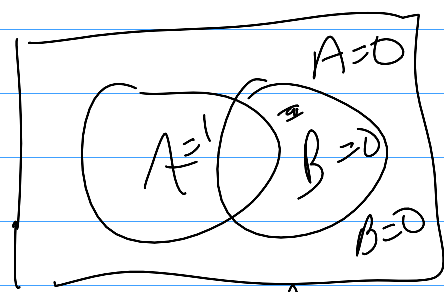
CD \ AB				
00	0	0	0	0
01	0	0	0	0
11	1	0	1	1
10	1	0	1	1

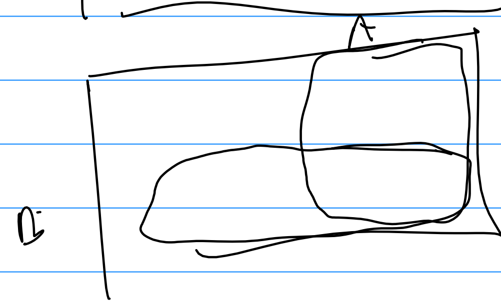
$h = g \cdot C$

$$\begin{bmatrix}1\\1\\1\\1\end{bmatrix} \cdot \begin{bmatrix}0\\0\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\\1\\1\end{bmatrix}$$

$A =$

CD \ AB				
	0	0	1	1
	0	0	1	1
	0	0	1	1
	0	0	1	1





C

AB

A

0	0	0	0
0	0	0	0
1	1	1	1
0	0	1	1

C

B

$f =$

A

0	0	0	0
0	0	0	0
1	0	1	1
0	0	0	0

C D

B

$f = h \cdot D$

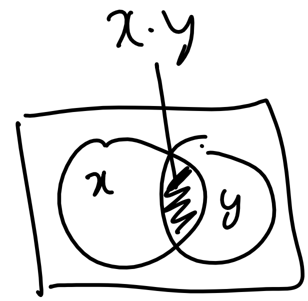
$= (A + \overline{B})\, C \cdot D$

Duality exists in axiom

3. $1 \cdot 1 = 1$
4. $0 + 0 = 0$
5. $0 \cdot 1 = 1 \cdot 0 = 0$
6. $\bar{0} = 1$
7. $\bar{1} = 0$
8. $x = 0$ if $x \neq 1$
9. $x = 1$ if $x \neq 0$

$\bar{0} = 1$

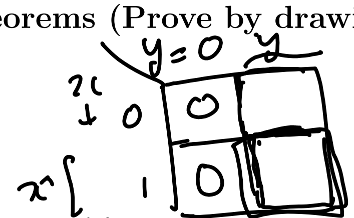
$\bar{1} = 0$



10.2 Single variable theorems (Prove by drawing K-maps)

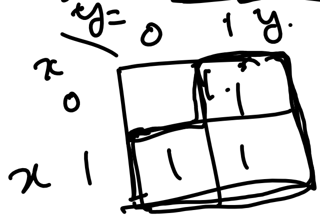
Duality
$\cdot \to +$
$0 \to 1$
$+ \to \cdot$
$1 \to 0$

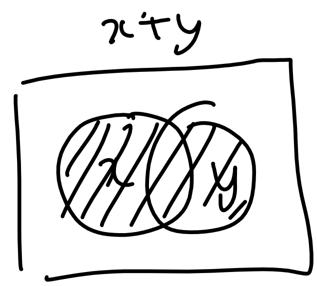
1. $x \cdot 0 = 0$



2. $x + 1 = 1$

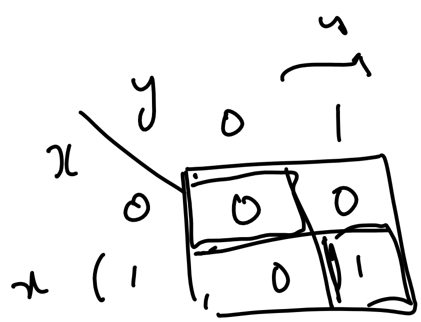
Th 2 is the dual of Th 1





3. $x \cdot 1 = x$

4. $x + 0 = x$



$x = 0$

5. $x \cdot x = x$

6. $x + x = x$

7. $x \cdot \bar{x} = 0$

8. $x + \bar{x} = 1$

9. $\bar{\bar{x}} = x$

Remark 2 (Duality). *Swap + with · and 0 with 1 to get another theorem*

10.3 Two and three variable properties (Prove by K-maps)

1. Commutative: $x \cdot y = y \cdot x$, $x + y = y + x$

2. Associative: $x \cdot (y \cdot z) = (x \cdot y) \cdot z$, $x + (y + z) = (x + y) + z$

$x \cdot y \cdot z$

Duals

$y + x \cdot z$

3. Distributive: $x \cdot (y + z) = x \cdot y + x \cdot z$, $\boxed{\cancel{x + y} \cdot z = (x + y) \cdot (y + z)}$

$$\text{RHS} = (x+y)\cdot(y+z) = x\cdot(y+z) + y\cdot(y+z)$$
$$= x\cdot y + x\cdot z + \underbrace{y\cdot y} + y\cdot z$$
$$= x\cdot y + x\cdot z + y + y\cdot z = y\cdot(\underbrace{x+1} + z) + x\cdot z$$
$$= y(\underbrace{1+z}) + x\cdot z$$
$$= y\cdot 1 + x\cdot z = y + x\cdot z$$

4. Absorption: $x + x \cdot y = x$, $x \cdot (x + y) = x$

$LHS = x + y \cdot z$

$z \backslash x \cdot y$				
	0	0	1	1
z	0	1	1	1

x, y

$RHS = (x + y) \cdot (y + z)$

$x + y$

$z \backslash xy$				
	0	1	1	1
z	0	1	1	1

x, y

$y + z =$

$z \backslash xy$				
	0	1	1	0
z	1	1	1	1

x, y

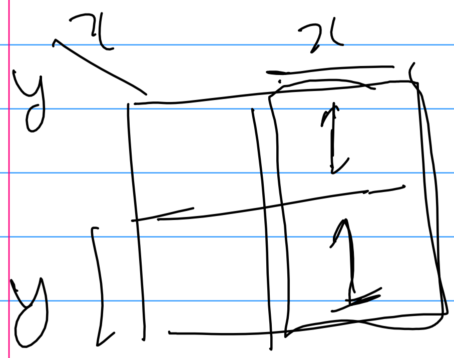
$z \backslash xy$				
	0	1	1	0
z	0	1	1	1

x, y

$y + x \cdot z$

$x + y \cdot x = x$

$x \cdot (y + x) = x$



LHS $= x + y \cdot x = x(1 + y)$

$= x \cdot 1$

$= x = \text{RHS}$

LHS $=$

$x(y + x) = x \cdot y + x \cdot x$

$= x \cdot y + x$

$= x(y + 1) = x \cdot 1 = x = \text{RHS}$

5. Combining: $\cancel{x \cdot y + x \cdot \bar{y}}$, $(x + y) \cdot (x + \bar{y}) = x$

$x \cdot y + x \cdot \bar{y} = x$

6. DeMorgan's theorem: $\overline{x \cdot y} = \bar{x} + \bar{y}$, $\overline{x + y} = \bar{x} \cdot \bar{y}$.

7. Concensus:

   (a) $x + \bar{x} \cdot y = x + y$

   (b) $x \cdot (\bar{x} + y) = x \cdot y$

   (c) $x \cdot y + y \cdot z + \bar{x} \cdot z = x \cdot y + \bar{x} \cdot z$

   (d) $(x + y) \cdot (y + z) \cdot (\bar{x} + z) = (x + y) \cdot (\bar{x} + z)$

De Morgan' Theorem

$$\overline{x \cdot y} = \bar{x} + \bar{y}$$

$$\overline{x \cdot y \cdot z} = \bar{x} + \bar{y} + \bar{z}$$

$$\overline{(x + y + z)} = \bar{x} \cdot \bar{y} \cdot \bar{z}$$

LHS $= \overline{x \cdot y}$

$x \cdot y =$

$y \backslash x$	$\bar{x}$	x
$\bar{y}$	0	0
y	0	1

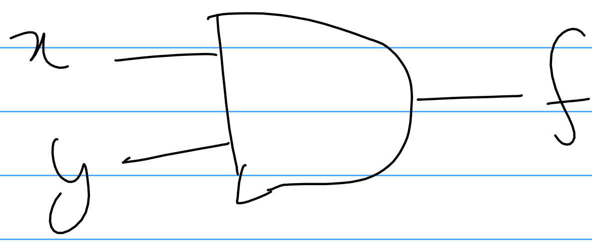
$\rightarrow$ $\overline{x \cdot y}$

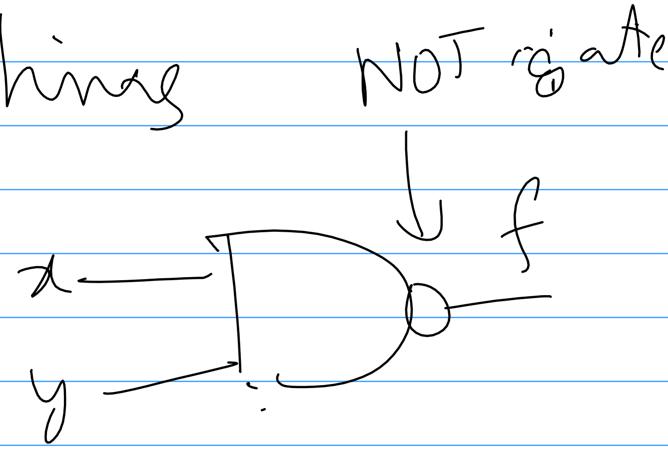
$y \backslash x$	$\bar{x}$	x
$\bar{y}$	1	1
y	1	0

RHS $= \bar{x} + \bar{y}$

$y \backslash x$	$\bar{x}$	x
$\bar{y}$	1	1
y	1	0

De Morgan Theorem as Bubble pushing

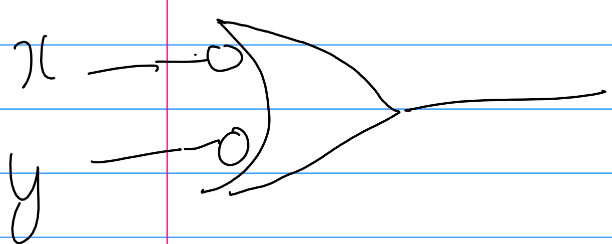


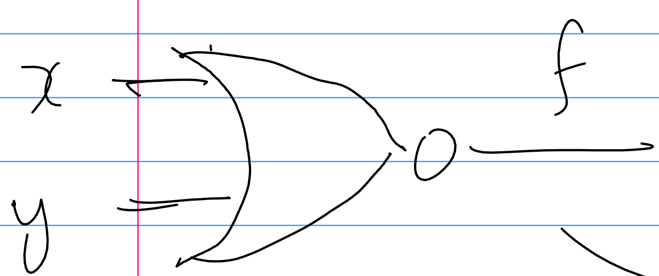


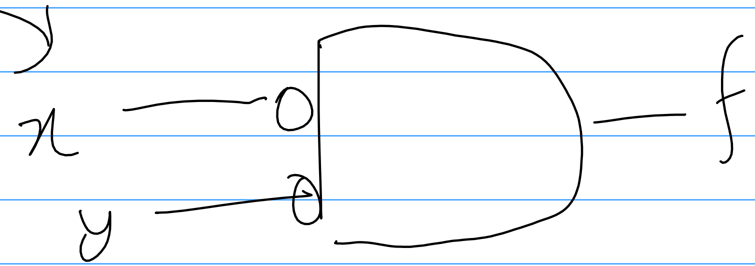
NAND

$$f = \overline{x \cdot y} = \bar{x} + \bar{y}$$

Bubble pushed through AND gate

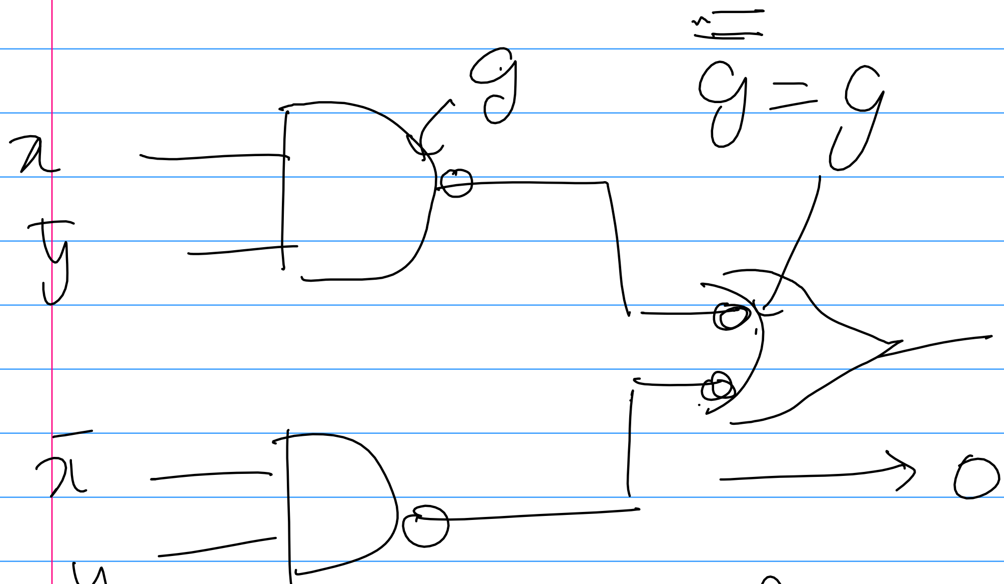




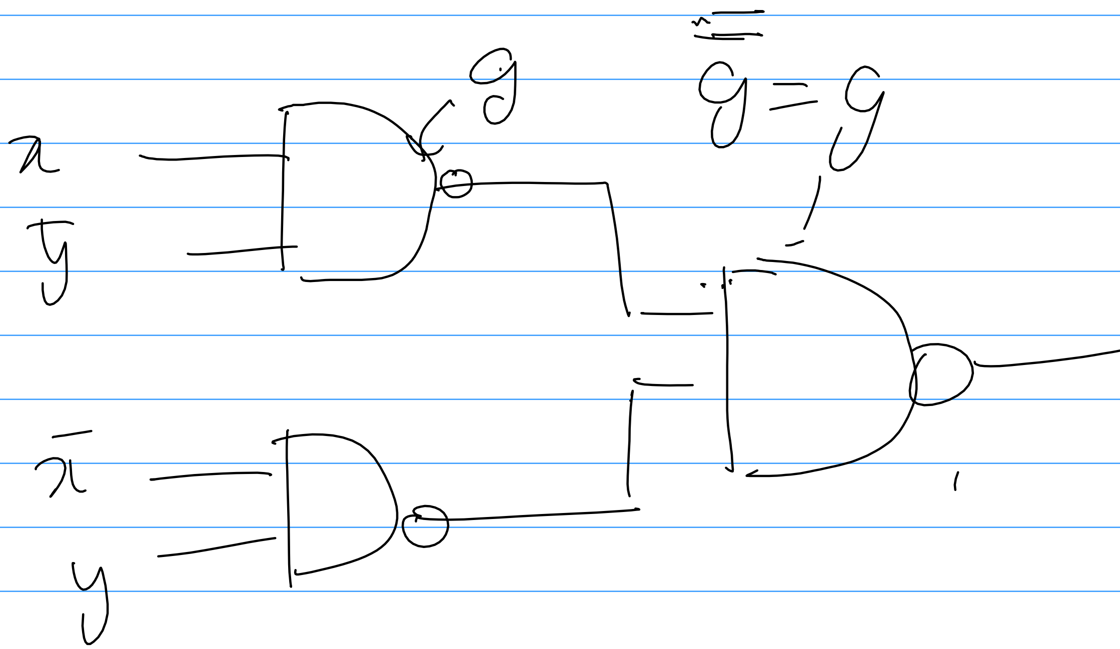


Sum of products form

$$f = x\bar{y} + \bar{x}y$$



AND-OR circuit



NAND-NAND circuit

$$LHS = x \cdot y + x \cdot \bar{y}$$
$$= x \cdot \underbrace{(y + \bar{y})}$$
$$= x \cdot 1$$
$$= x \cdot = RHS$$

Dual

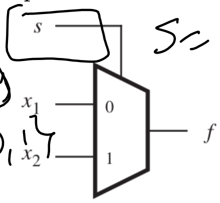
$$LHS = (x + y)(x + \bar{y})$$
$$= \underbrace{x \cdot x} + x \cdot \bar{y} + y \cdot x + y \cdot \bar{y}$$
$$= x + x \cdot \bar{y} + y \cdot x + \underbrace{y \cdot \bar{y}}_{0}$$
$$= x + x\bar{y} + y \cdot x + 0$$
$$= x + x(y + \bar{y})$$
$$= x + x \cdot 1$$
$$= x + x$$
$$= x$$

Example 11 (Multiplexer)**.** *Multiplexer is a circuit used to select one of the input lines x_1 and x_2 based only select input s. When $s = 0$, x_1 is selected, x_2 is selected otherwise. Find a boolean expression and a circuit for multiplexer*

$s = 0$

$x_1 \in \{0, 1\}$

$x_2 \in \{0, 1\}$



s	$f(s, x_1, x_2)$
0	x_1
1	x_2

s	x_1	x_2	f
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Example 12. *Simplify $f = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}\bar{C}$ using boolean algebra*

Example 13. *Simplify $f = \bar{A}\bar{A}\bar{C} + \bar{A}\bar{B}C$ using K-maps.*

$s, x_1, x_2 \rightarrow$

	s	x_1	x_2	f
0	0	0	0	0
1	0	0	1	0
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	0
7	1	1	1	1

$s x_1 \rightarrow$, $x_2 \downarrow$

x_2 \ $s x_1$	0	2	6	4
	0	1	0	0
x_2	0 (1)	1 (3)	1 (7)	1 (5)

S over columns 6, 4; x_1 under columns 3, 7.

$s x_1 \rightarrow$ 00 01 10 11, $x_2 \downarrow$

	00	01	10	11

$x_1 x_2 \rightarrow$, $s \downarrow$

	0	1		2
s				

x_1 over the upper columns; x_2 under the lower columns.

x_1	x_2	s	f

f

$s x_1 \rightarrow$, $x_2 \downarrow$

x_2 \ $s x_1$	$\bar{s}$: 0	2	S: 6	4
	0	1	0	0
x_2	0 (1)	1 (3)	1 (7)	1 (5)

$\bar{S}'$ x_1

✓ $f = m_2 + m_3 + m_5 + m_7$

✓ $f = x_1 \cdot \bar{S}$

$+ x_2 x_1 \leftarrow$ unnecessary

$+ x_2 \cdot S$

$f = x_1 \cdot \bar{S} + x_2 S$