Logic minimization: Minimum-cost circuits

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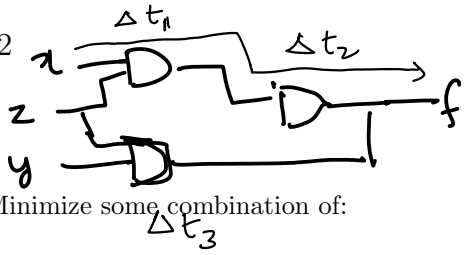
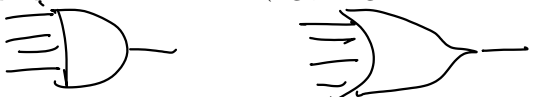


1 Logic minimization

A general optimization criteria for multi-level logic are to Minimize some combination of:

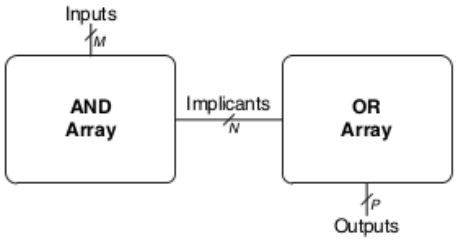
1. Area occupied by the logic gates and interconnect;
2. the Critical Path Delay of the longest path through the logic;
3. the Degree of Testability of the circuit, measured in terms of the percentage of faults covered by a specified set of test vectors, for an appropriate fault model (Eg., single stuck faults, multiple stuck faults, etc.);
4. Power consumed by the logic gates.

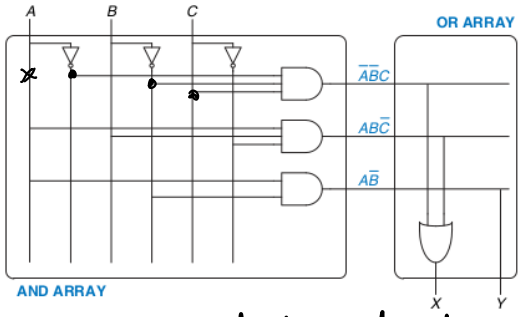


In this course, we will start with two-level multi-input circuits and a criteria based on the number of gates/transistors/diodes.

2 Programmable Logic Arrays (PLA)

Two level circuit





sum of products form

3 Two-level circuits

The cost that we are going to consider in this class depend upon:

1. Number of gates.
2. Number of input to the gates.

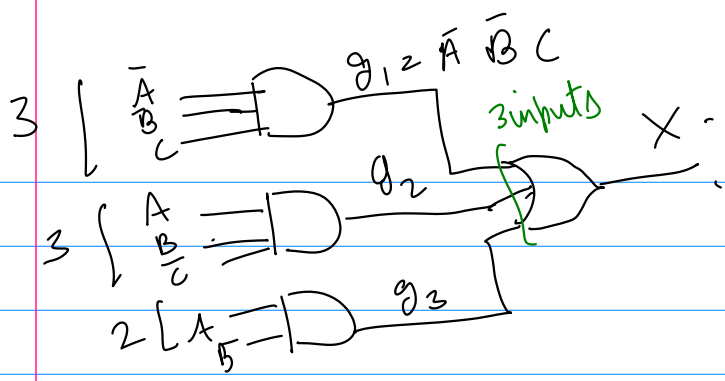
More gates need more transistors, more area on the chip. More-inputs the gate need more transistors within each gate. Number of gate inputs can be considered secondary criterion to the number of gates.

Example 1. *Find the cost of the following Boolean expression* $X = \bar{A}\bar{B}C + AB\bar{C} + A\bar{B}$.

1 and + 1 and + 1 and = 3 and gates } 4 gates

Problem 1. *Find the cost of the following Boolean expression* $X = A\bar{B}C + \bar{A}B\bar{C} + \bar{B}C$.

= 1 OR gate



$X = g_1 + g_2 + g_3$

4 gates

Total Number of inputs

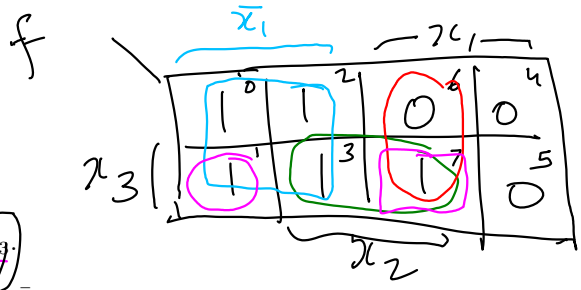
$= \underbrace{3+3+2}_{\text{AND}} + \underbrace{3}_{\text{OR}}$

$= 8+3 = 11$

Total cost $= 4 + 11 = 15$

OR = Union of regions

$f = m_0 + m_1 + m_2 + m_3 + m_7$



4 Terminology for K-maps

Running Example: $f = \sum m(0,1,2,3,7) = \bar{x}_1 + x_1 x_2 x_3$.

Literal A single variable or its complement. Example: $\bar{x}, x_1, x_2, x_3$

Implicant A product term which is true for a function. All minterms are implicants. Example: $x_1x_2x_3$, $\bar{x}_1$, $m_0 = \bar{x}_1\bar{x}_2\bar{x}_3$, $\bar{x}_1x_3$, $\bar{x}_1\bar{x}_3$.

Product Term

x_1, x_2 × x_2x_3 ✓ $\bar{x}_1$ ✓

Prime Implicant An implicant that cannot be combined into fewer literals. Example: $\bar{x}_1, x_2x_3$.

Essential Prime Implicant An implicant that cannot be combined into fewer literals. Example: x_2x_3.

$m_1 = \bar{x}_1 x_2 x_3$ ✓

Cover : List of Prime Implicants that account for all $f = 1$.

Cost : Number of gates (excluding not gate on literals) and number of inputs to each gate.

SOP form

Example 2. *Find minimum cost expression for the function* $f(x_1, x_2, x_3) = \prod M(4,5,6)$

Min. cost expression from K-map

1. Find all PI
2. Find all EPI
3. $f = \sum$ EPI + ____
4. Do EPIs cover all $f=1$

if (4) is false

Petrick's methods

Problem 2. *Find minimum cost expression for the function* $f(x_1, x_2, x_3) = \prod M(2,5,6)$

4.1 Incompletely specified functions or Don't cares

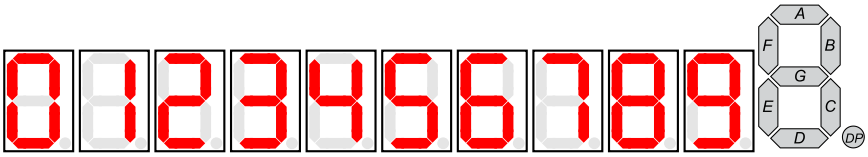
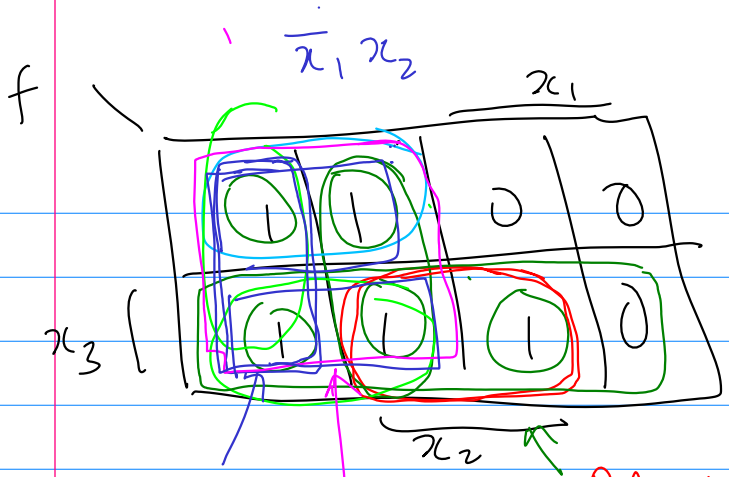
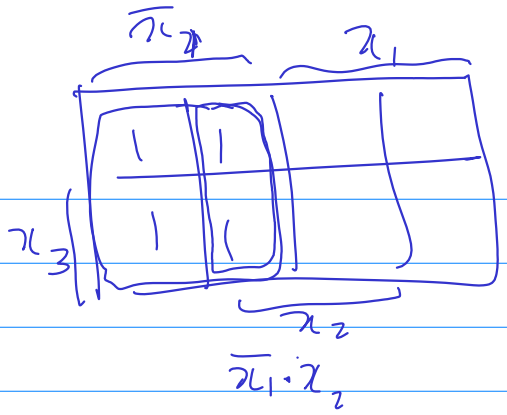


Figure 1: 7 Segment Representations of Each Integer



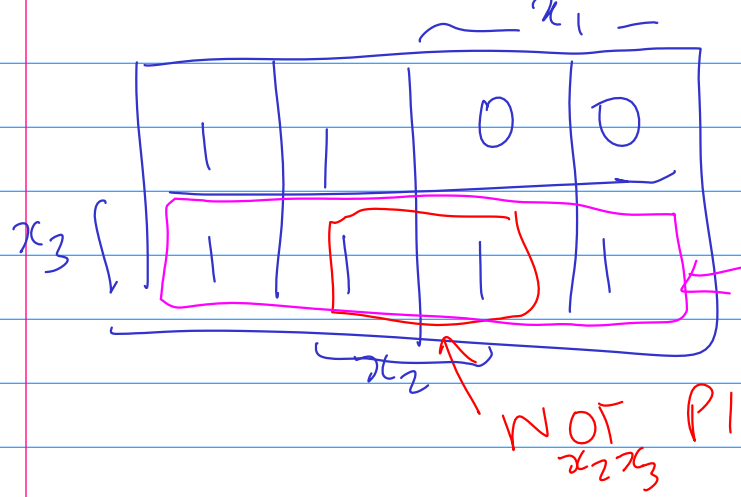


NOT PI
$= \bar{x}_1 \bar{x}_2$

PI $= \bar{x}_1$

PRIME IMPLICANT (fewest literals)
$= x_2 x_3$

= fewest inputs
= lowest cost



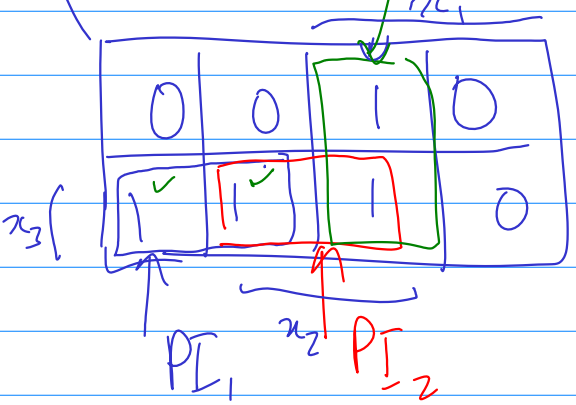
$PI = x_3$

NOT PI
$x_2 x_3$

EPI

lowers = 2, 4, 8, 16, 32 of 2

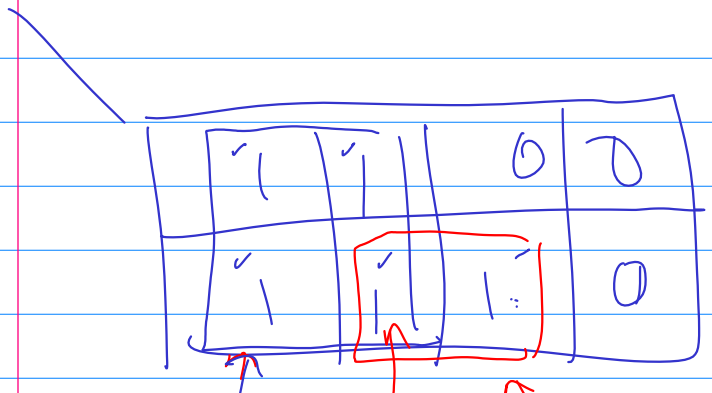
PI_3



PI_1

PI_2

EPI_1



Essential PI
EPI

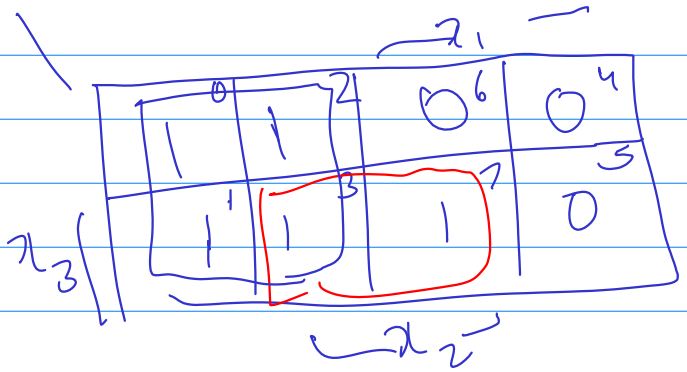
Essential PI
EPI

$$f(x_1, x_2, x_3) = \prod M(4, 5, 6)$$

Maxterms

x_1	x_2	x_3	f
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

f



① PIs
$\bar{x}_1, x_2 x_3$

② EPIs
$\bar{x}_1, x_2 x_3$

③ $f = \bar{x}_1 + x_2 x_3$

1 AND #2

1 OR gate #2

④ Stop

Cost of f = 2 gates + 4 inputs

= 6

BCD Value				LED Segment
D_3	D_2	D_1	D_0	E
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	d
1	0	1	1	d
1	1	0	0	d
1	1	0	1	d
1	1	1	0	d
1	1	1	1	d

Example 3. *Find minimum cost expression for the function*

$$f(x_1, \ldots, x_4) = \sum m(2, 4, 5, 6, 10) + D(12, 13, 14, 15)$$

Problem 3. *Find minimum cost expression for the function*

$$f(x_1, \ldots, x_4) = \sum m(0, 2, 4, 6, 7, 8, 9, 13) + D(1, 12, 15)$$