

Logic minimization: Minimum-cost circuits

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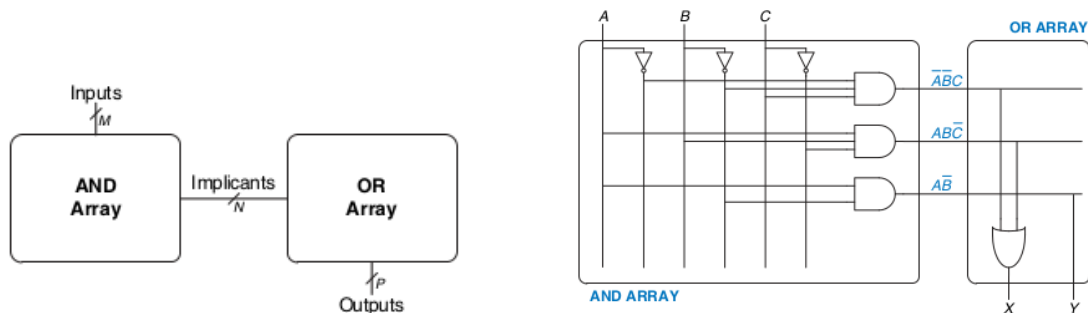
1 Logic minimization

A general optimization criteria for multi-level logic are to Minimize some combination of:

1. Area occupied by the logic gates and interconnect;
2. the Critical Path Delay of the longest path through the logic;
3. the Degree of Testability of the circuit, measured in terms of the percentage of faults covered by a specified set of test vectors, for an appropriate fault model (Eg., single stuck faults, multiple stuck faults, etc.);
4. Power consumed by the logic gates.

In this course, we will start with two-level multi-input circuits and a criteria based on the number of gates/transistors/diodes.

2 Programmable Logic Arrays



3 Two-level circuits

The cost that we are going to consider in this class depend upon:

1. Number of gates.
2. Number of input to the gates.

More gates need more transistors, more area on the chip. More-inputs the gate need more transistors within each gate. Number of gate inputs can be considered secondary criterion to the number of gates.

Example 1. Find the cost of the following Boolean expression $X = \bar{A}\bar{B}C + AB\bar{C} + A\bar{B}$.

Problem 1. Find the cost of the following Boolean expression $X = A\bar{B}C + \bar{A}B\bar{C} + \bar{B}C$.

4 Terminology for K-maps

Running Example: $f = \sum m(0, 1, 2, 3, 7) = \bar{x}_1 + x_1x_2x_3$.

Literal A single variable or its complement. Example: \bar{x}, x_1, x_2, x_3

Implicant A product term which is true for a function. All minterms are implicants. Example: $x_1x_2x_3, \bar{x}_1, m_0 = \bar{x}_1\bar{x}_2\bar{x}_3, \bar{x}_1x_3, \bar{x}_1\bar{x}_3$.

Prime Implicant An implicant that cannot be combined into fewer literals. Example: \bar{x}_1, x_2x_3 .

Essential Prime Implicant An implicant that cannot be combined into fewer literals. Example: x_2x_3 .

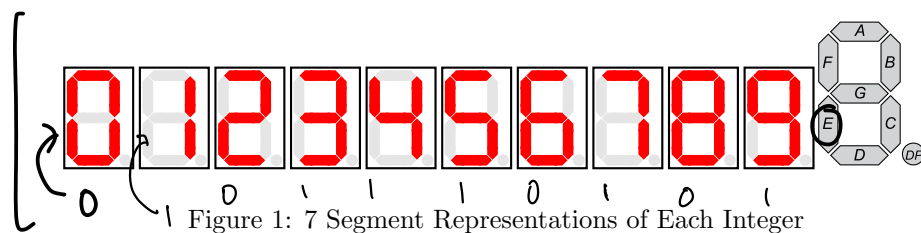
Cover : List of Prime Implicants that account for all $f = 1$.

Cost : Number of gates (excluding not gate on literals) and number of inputs to each gate.

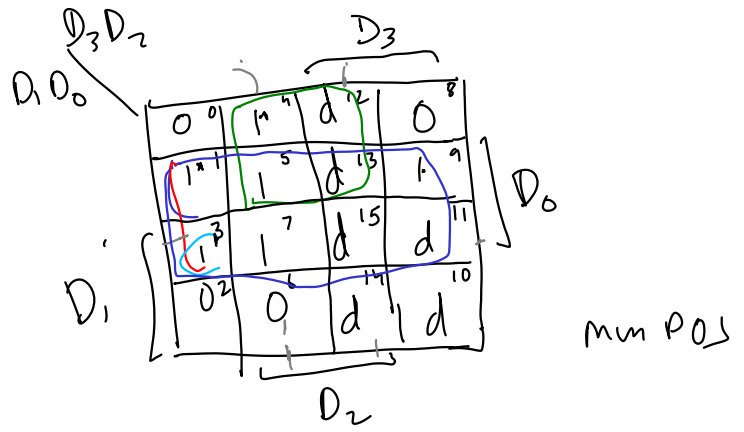
Example 2. Find minimum cost expression for the function $f(x_1, x_2, x_3) = \prod M(4, 5, 6)$

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4.1 Incompletely specified functions or Don't cares



	BCD Value				LED Segment
	D_3	D_2	D_1	D_0	E
0	0	0	0	0	0
1	0	0	0	1	1
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	1
10	1	0	1	0	x d
11	1	0	1	1	d
12	1	1	0	0	d
13	1	1	0	1	d
14	1	1	1	0	d
15	1	1	1	1	d



Don't care

Min SOP

$$EPIs = D_0, D_2, \bar{D}_1$$

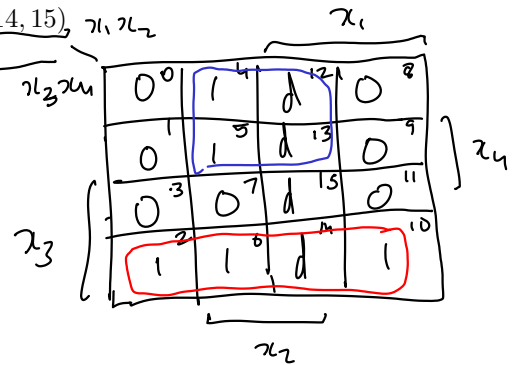
$$f_E(D_3, \dots, D_0) = D_0 + D_2 \bar{D}_1$$

Example 3. Find minimum cost expression for the function

$$f(x_1, \dots, x_4) = \sum m(2, 4, 5, 6, 10) + D(12, 13, 14, 15)$$

min SOP

$$f = \underline{x_3 \bar{x}_1} + \underline{x_2 \bar{x}_3}$$



Problem 3. Find minimum cost expression for the function

$$f(x_1, \dots, x_4) = \sum m(0, 2, 4, 6, 7, 8, 9, 13) + D(1, 12, 15)$$