NAND/NOR gates + Quine McCluskey + Petricks

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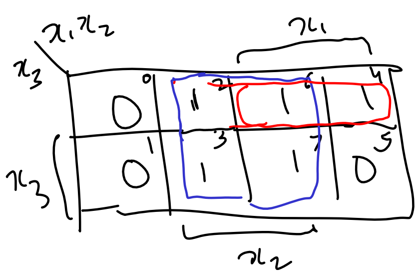
1 Circuit design using NAND/NOR gates

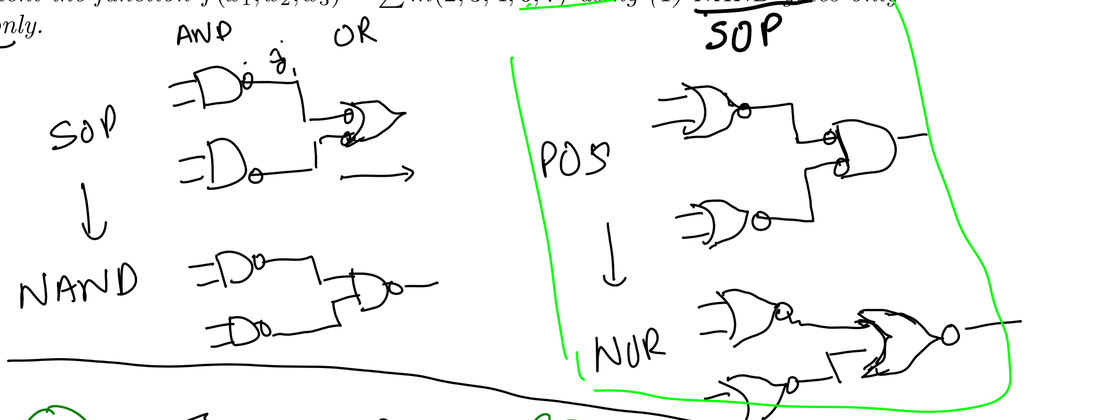
Example 1. *Implement the function $f(x_1, x_2, x_3) = \sum m(2, 3, 4, 6, 7)$ using (1) NAND gates only and (2) NOR gates only.*

minterms

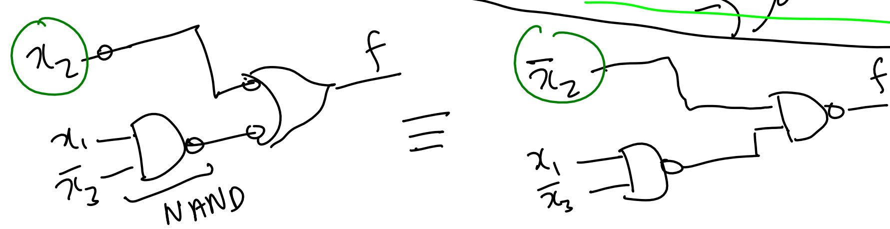
min SOP

POS



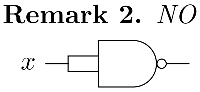


$f = x_2 + x_1\bar{x}_3$

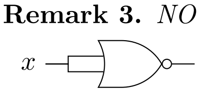


Remark 1. *NAND-NAND logic is generated from SOP form. NOR-NOR logic is generated from POS form.*

Remark 2. *NOT gate can also be created from a NAND gate $\bar{x} = \overline{x \cdot x}$.*



Remark 3. *NOT gate can also be created from a NOR gate $\bar{x} = \overline{x + x}$.*



Problem 1. *Design the simplest circuit that implements the function $f(x_1, x_2, x_3) = \sum m(3, 4, 6, 7)$ using (1) NAND gates only (2) NOR gates only.*

2 Quine-McCluskey

This is not in the text-book. For additional reading, please refer to the linked resources on the website.

Definition 1 (Implicant)**.** *Given a function f of n variables, a product term P is an implicant of f if and only if for every combination of values of the n variables for which $P = 1$, f is also equal to 1.*

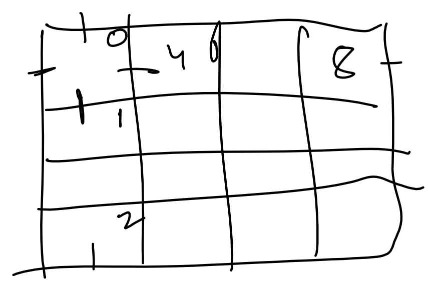
Definition 2 (Prime Implicant)**.** *A prime implicant of a function f is an implicant which is no longer an implicant if any literal is removed from it.*

There are 4 main steps in the Quine-McCluskey algorithm:

1. Generate Prime Implicants
2. Construct Prime Implicant Table. PIs as columns, and minterms as rows (don't cares are excluded).
3. Reduce Prime Implicant Table by repeating following steps until they it cannot be reduced further
   (a) Remove Essential Prime Implicants
   (b) Row Dominance: Remove *dominating* rows. (i.e. unnecessary minterms)
   (c) Column Dominance: Remove *dominated* columns. (i.e. remove unnecessary PIs)
4. Solve Prime Implicant Table by Petricks method

2.1 Generate Prime Implicants

Example 2. *Generate prime implicants of the function $F(A, B, C, D) = \sum m(0, 2, 5, 6, 7, 8, 10, 12, 13, 14, 15)$ using Quine-McCluskey method*



Steps:

1. Start with writing minterms in binary format (include don't cares as minterms).
2. Create potential groups of minterms that can be combined (merged). The only minterms that can be combined differ only be single 1. Create a new list of combined minterms as n-1 literal implicants.

3. Check off the minterms that could be combined. Unchecked minterms are prime implicants (PIs).

4. Repeat the grouping process with n-1 literal implicants.

Problem 2. *Generate PIs for the function* $F(A, B, C, D) = \sum m(0, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13)$.

2.2 Prime Implicants table and reduction

Example 3. *Reduce the prime implicants* $\{\bar{B}\bar{D}, C\bar{D}, BD, BC, A\bar{D}, AB\}$ *using prime implicants table.*

Example 4.

$CD \backslash AB$	00	01	11	10
00	1	1	0	0
01	0	1	1	0
11	0	0	1	1
10	0	0	0	0

Example 5.

$CD \backslash AB$	00	01	11	10
00	d	0	0	0
01	1	1	d	d
11	1	1	0	0
10	1	d	0	0

Example 6. *Reduce the following PI table*

	$\bar{A}\bar{D}$	$\bar{B}\bar{D}$	$\bar{C}\bar{D}$	$\bar{A}C$	$\bar{B}C$	$\bar{A}B$	$B\bar{C}$	$A\bar{B}$	$A\bar{C}$
0	X	X	X						
2	X	X		X	X				
3				X	X				
4	X		X			X	X		
5						X	X		
6	X			X		X			
7				X		X			
8		X	X					X	X
9								X	X
10		X			X			X	
11					X			X	
12			X		X		X		X
13							X		X

2.3 Petricks method

Example 7. *Solve the Prime Implicant table using Petrick's method*

	$p_1 = \bar{A}C$	$p_2 = \bar{B}C$	$p_3 = \bar{A}B$	$p_4 = B\bar{C}$	$p_5 = A\bar{B}$	$p_6 = A\bar{C}$
3	X	X				
5			X	X		
7	X		X			
9					X	X
11		X			X	
13				X		X

Example 8. *Find the minimum SOP expression for the function* $F(A, B, C, D) = \sum m(2, 3, 7, 9, 11, 13) + \sum d(1, 10, 15)$ *using Quine-McCluskey method.*

min POS

1. $\bar{f}$
2. min SOP for $\bar{f}$
3. Take inverse on both sides
4. Apply Demorgan's to get PO

min SOP POS

f

$x_3 \backslash x_1x_2$	00	01	11	10
0	0	1	1	1
1	0	1	1	0

$\bar{f}$

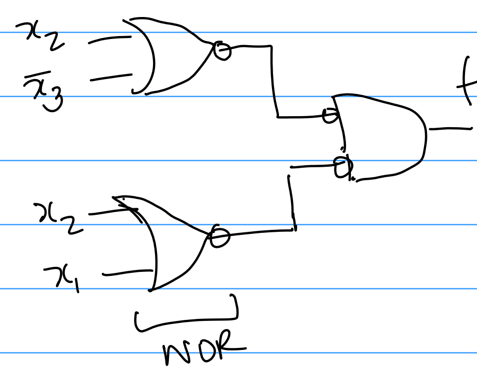
$x_3 \backslash x_1x_2$	00	01	11	10
0	1	0	0	0
1	1	0	0	1

$$\bar{f} = \bar{x}_2 x_3 + \bar{x}_2 \bar{x}_1$$

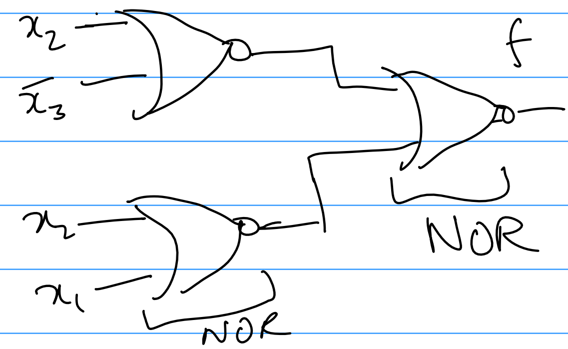
$$\Rightarrow \bar{\bar{f}} = \overline{\bar{x}_2 x_3 + \bar{x}_2 \bar{x}_1}$$

$$\Rightarrow f = \overline{\bar{x}_2 x_3} \cdot \overline{\bar{x}_2 \bar{x}_1}$$

$$= (x_2 + \bar{x}_3) \cdot (x_2 + x_1) \leftarrow \text{min POS}$$



$\Rightarrow$



Quine McCluskey

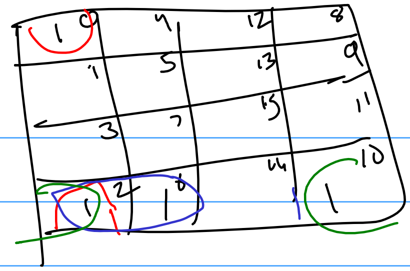
Group #	Minterms	Representation
0	0	0000
1	1	0001
	2	0010
	4	0100
	8	1000
2	3	0011
	5	0101
	6	0110
	9	1001
	10	1010
	12	1100
3	7	0111
	11	1011
	13	1101
	14	1110
4	15	1111

$F(A,B,C,D) = \sum m(0,2,5,6,7,8,10,12,13,14,15)$ ✓

Group #	Minterms	Repr
0	0	0000 ✓
1	2	0010 ✓
	8	1000 ✓
2	5	0101 ✓
	6	0110 ✓
	10	1010 ✓
	12	1100 ✓
3	7	0111 ✓
	13	1101 ✓
	14	1110
4	15	1111

3-literal implicant

Minterms	Representation
(0,2)	00-0
(0,8)	-000
(2,6)	0-10
(2,10)	-010
(8,10)	10-0
(8,12)	1-00
(5,7)	01-1
(5,13)	-101
(6,7)	01-1
(6,14)	-110
(10,14)	1-10
(12,13)	110-
(12,14)	11-0



$\bar{A}\bar{B}\bar{D}$

Group #	Minterms	Repr
0	0	0000 ✓
1	2	0010 ✓
	8	1000 ✓
2	5	0101 ✓
	6	0110 ✓
	10	1010 ✓
	12	1100 ✓
3	7	0111 ✓
	13	1101 ✓
	14	1110 ✓
4	15	1111 ✓

3-literal implicant

Minterms	Represent
? (0,2)	00-0 ✓
(0,8)	-000 ✓
(2,6)	0-10 ✓
(2,10)	-010 ✓
? (8,10)	10-0 ✓
(8,12)	1-00 ✓
(5,7)	01-1 ✓
(5,13)	-101 ✓
(6,7)	011- ✓
(6,14)	-110 ✓
(10,14)	1-10 ✓
(12,13)	110- ✓
12,14	11-0 ✓
(7,15)	-111 ✓
(13,15)	11-1 ✓
(14,15)	111- ✓

2-literal implicants

Minterms	Representation
(0,2,8,10)	-0-0
~~(0,8,2,10)~~	~~-0-0~~
(2,6,10,14)	--10
~~2,10,6,14)~~	~~--10~~
(8,10,12,14)	1--0
~~(8,12,10,14)~~	~~1--0~~
(12,13,14,15)	11--

$\bar{B}\bar{D}$

① Dash should match exactly
② exactly 1-bit should differ

on the implicant should differ by exactly 1-bit

(5, 7, 13, 15)	-1-1
~~(5, 13, 7, 15)~~	-1-1
(6, 7, 14, 15)	-11-
~~(6, 14, 7, 15)~~	~~-11-~~
~~(12, 14, 13, 15)~~	

2-literal implicants

Minterms	Repr
(0, 2, 8, 10)	$-0-0 = \bar{B}\bar{D}$
(2, 6, 10, 14)	$--10 = C\bar{D}$
(8, 10, 12, 14)	$1--0 = A\bar{D}$
(12, 13, 14, 15)	$11-- = AB$
(5, 7, 13, 15)	$-1-1 = BD$
(6, 7, 14, 15)	$-11- = BC$

1-literal implicant

X

$PIs = \{\bar{B}\bar{D}, C\bar{D}, A\bar{D}, AB, BD, BC\}$

Verilog: find minimum cost expression is NP-complete / NP-hard

computation time $\propto \exp(n)$

→ approximation

PI table

minterms ↓ / PIs →

$\bar{B}\bar{D}$	$C\bar{D}$	BD	BC	$A\bar{D}$	AB
-0-0	--10	-1-1	-11-	1--0	11--
0000 = 0 0010 = 2 1000 = 8 1010 = 10 (0, 2, 8, 10)	(2, 6, 10, 14)	(5, 7, 13, 15)	(6, 7, 14, 15)	(8, 10, 12, 14)	(12, 13, 14, 15)

PI table

PIs →

minterms ↓

minterms	$\bar{B}\bar{D}$	$C\bar{D}$	BD	BC	$A\bar{D}$	AB
	$-0-0$ $0000 = 0$ $0010 = 2$ $1000 = 8$ $1010 = 10$ $(0,2,8,10)$	$--10$ $(2,6,10,14)$	$-1-1$ $(5,7,13,15)$	$-11-$ $(6,7,14,15)$	$1--0$ $(8,10,12,14)$	$11--$ $(12,13,14,15)$
0*	X					
2	X	X				
5*			X			
6		X		X		
7			X	X		
8	X				X	
10	X	X			X	
12					X	X
13			X			X
14		X		X	X	X
15			X	X		X

(1) Find EPIs = $\{\bar{B}\bar{D}, BD)$