

# Homework 4

Max marks: 40

Due on Oct 1st, 2021, 9 AM, before class.

**Problem 1** Hazard problem: Design a hazard free SOP for  $f(A, B, C, D) = \sum m(0, 1, 4, 5, 6, 7, 9, 11, 14, 15)$

$$f = hx_1 + \bar{h}\bar{x}_1 \quad (1)$$

$$= \bar{x}_1 \oplus h \quad (2)$$

$$= \bar{x}_1 \oplus (x_3 \oplus x_4) \quad (3)$$

**Solution**

The K-map for  $f$  is

		$\bar{A}$	$A$	
		$\bar{B}$	$B$	$\bar{B}$
$\bar{C}$	$\bar{D}$	1	1	0
	$D$	1	1	0
$C$		0	1	1
	$\bar{D}$	0	1	1

$$f = \bar{A}B + BC + ACD + \bar{A}\bar{C} + \bar{A}\bar{B}D + \bar{B}\bar{C}D$$

Cost( $f$ ) = 6 AND gates + 2\*3 + 3\*3 inputs to AND gates + 1 OR gate + 6 inputs = 28.

**Problem 2** Find the simplest realization of the function  $f(x_1, \dots, x_4) = \sum m(0, 3, 4, 7, 9, 10, 13, 14)$ , assuming that the logic gates have a maximum fan-in of two.

**Solution**

The K-map for  $f$  is

		$\bar{x}_1$	$x_1$	Row pattern
		$\bar{x}_2$	$x_2$	$\bar{x}_2$
$\bar{x}_3$	$\bar{x}_4$	1	1	0
	$x_4$	0	0	1
$x_3$		1	1	0
	$\bar{x}_4$	0	0	1
Col pattern		$\bar{h}$	$\bar{h}$	$h$

Writing  $f$  in terms of column pattern  $h = \bar{x}_3x_4 +$

Cost( $f$ ) = 2 XOR gates + (2+2) inputs each = 6. Max fan-in = 2.

**Problem 3** Find the minimum-cost circuit for the function  $f(x_1, \dots, x_4) = \sum m(0, 4, 8, 13, 14, 15)$ . Assume that the input variables are available in uncomplemented form only. (Hint: Use functional decomposition.)

**Solution**

The K-map for  $f$  is

		$\bar{x}_1$	$x_1$	Row pattern
		$\bar{x}_2$	$x_2$	$\bar{x}_2$
$\bar{x}_3$	$\bar{x}_4$	1	1	0
	$x_4$	0	0	1
$x_3$		0	0	1
	$\bar{x}_4$	0	0	1
Col pattern		$h = \bar{x}_3\bar{x}_4$	$h$	$\bar{h}$

Write  $f$  in terms of  $g$ ,

$$f = \bar{x}_3\bar{x}_4\bar{g} + \bar{x}_3\bar{x}_4g \quad (4)$$

$$= \bar{x}_3 + \bar{x}_4\bar{g} + \underbrace{(x_3 + x_4)}_{h_2}g \quad (5)$$

$$= \bar{h}_2\bar{g} + h_2g \quad (6)$$

$$= \bar{h}_2 \oplus g \quad (7)$$

$$= (\bar{x}_3 + x_4) \oplus (x_1x_2) \quad (8)$$

Cost( $f$ ) = 1 OR gate + 2 inputs for OR gate + 1 AND gate + 2 inputs for AND gate + 1 XOR gate + 2 inputs for XOR gate + 1 NOT gate + 1 input to NOT gate = 11.

		$\bar{x}_1$				Row pattern	$x_1$				Row pattern
		$\bar{x}_2$	$x_2$				$\bar{x}_2$	$x_2$			
		$\bar{x}_3$	$x_3$	$\bar{x}_3$			$\bar{x}_3$	$x_3$	$\bar{x}_3$		
$\bar{x}_4$	$\bar{x}_5$	d	0	0	0	<b>0</b>	0	d	0	0	<b>0</b>
	$x_5$	1	0	0	1	$\bar{g} = \bar{x}_3$	0	0	0	1	$g_2 = x_2\bar{x}_3$
$x_4$		0	1	d	0	$g = x_3$	1	0	1	0	$g_3 = \bar{x}_2 \oplus x_3$
	$\bar{x}_5$	1	0	0	1	$\bar{g} = \bar{x}_3$	1	0	0	d	$g = \bar{x}_3$
Col pattern		$h = x_4 \oplus x_5$	$h_2 = x_4x_5$	<b>0</b>	$h$		$h_3 = x_4$	<b>0</b>	$h_2 = x_2x_4$	$h$	

Table 1: K-map for Problem 4.

		$\bar{x}_1$				$x_1$			
		$\bar{x}_2$	$x_2$			$\bar{x}_2$	$x_2$		
		$\bar{x}_3$	$x_3$	$\bar{x}_3$		$\bar{x}_3$	$x_3$	$\bar{x}_3$	
$\bar{x}_4$	$\bar{x}_5$	d	0	0	0	0	d	0	0
	$x_5$	1	0	0	1	0	0	0	1
$x_4$		0	1	d	0	1	0	1	0
	$\bar{x}_5$	1	0	0	1	1	0	0	d

Table 2: K-map for Problem 4.

**Problem 4** Use functional decomposition to find the best implementation of the function  $f(x_1, \dots, x_5) = \sum m(1, 2, 7, 9, 10, 18, 19, 25, 31) + D(0, 15, 20, 26)$ . How does your implementation compare with the lowest-cost SOP implementation? Give the costs.

**Solution**

The K-map for  $f$  is given in Table 1.

We try writing  $\bar{x}_1 = 1$  half of  $f$  in terms of Row patterns  $g = x_3$ , and  $x_1 = 1$  half of  $f$  in terms normal K-map grouped terms,

$$\begin{aligned} f &= \bar{x}_1 \left( \underbrace{(\bar{x}_4 x_5 + x_4 \bar{x}_5)}_h \bar{x}_3 + \underbrace{x_4 x_5}_{h_2} x_3 \right) \\ &\quad + x_1 \bar{x}_2 \bar{x}_3 x_4 + x_2 x_3 h_2 + x_2 \bar{x}_3 h \\ &= \bar{x}_1 h \bar{x}_3 + \bar{x}_1 h_2 x_3 + x_1 \bar{x}_2 \bar{x}_3 x_4 + x_2 x_3 h_2 \\ &\quad + x_2 \bar{x}_3 h \end{aligned}$$

Cost( $h = x_4 \oplus x_5$ ) = 1+2=3. Cost( $h_2$ ) = 1+2=3. Total Cost of  $f$  = Cost( $h$ ) + Cost( $h_2$ ) + 5 AND gates + 4\*3 input per gate + 4 inputs to AND gates + 1 OR gate + 5 inputs = 3+3+5 + 12+4+1+5=33.

Normal grouping is shown in Table 2.

$$\begin{aligned} f &= \bar{x}_3 x_4 \bar{x}_5 + \bar{x}_1 \bar{x}_3 \bar{x}_4 \bar{x}_5 + \bar{x}_1 x_3 x_4 x_5 \\ &\quad + x_1 \bar{x}_2 \bar{x}_3 x_4 + x_2 x_3 x_4 x_5 + x_2 \bar{x}_3 \bar{x}_4 x_5 \quad (9) \end{aligned}$$

Cost( $f$ ) = 6 AND gates +(3+5\*4 inputs to AND gates)+1 OR gate+6 inputs to the OR gate=36.