ECE 417/598: Camera calibration

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[1] See Hartley and Zisserman's Multiple View Geometry for details.

Announcements

- Midterm next Friday.
- Will cover everything until today's lecture
- I will release Sample exam on Monday
- No programming questions. Expect Linear Algebra question though. Only the concepts that we have touched in class.
- Grading: compare you answers to the solution.

$$u \approxeq v(\epsilon) \equiv |u - v| \leq \epsilon \min(|u|, |v|) \tag{1}$$

$$u \sim v(\epsilon) \equiv |u - v| \leq \epsilon \max(|u|, |v|) \tag{2}$$

Homogeneous representation of lines

$\mathbb{R}$

$\underline{x}_i = \lambda \underline{x}_j$

$\underline{l}_i = k \underline{l}_j$

(k: scalar)

$$ax + by + c = 0$$

$a(\lambda x) + b(\lambda y) + c(\lambda 1) = 0$

$ax + by + c \cdot 1 = 0$

$(ka)x + (kb)y + (kc)1 = 0$

$$\underbrace{\begin{bmatrix} a & b & c \end{bmatrix}}_{\underline{l}} \underbrace{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}}_{\underline{x}} = 0$$

$a:b$

$b:c$

$\underline{l}^T \underline{x} = 0$

Homogeneous line $= \underline{l}$

$\underline{l}^T \underline{x} = 0$

Projective space

$\mathbb{P}^2$

2D-

$$\mathbb{P}^2 = \mathbb{R}^3 - \{(0,0,0)^\top\}$$

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad \underline{x}_i = \lambda \underline{x}_j \qquad \lambda \neq 0$$

$$\mathbb{P}^3 = \mathbb{R}^4 - \{(0,0,0,0)^\top\}$$

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \qquad \underline{x}_i = \lambda \underline{x}_j$$

Homogeneous representation of points

$$ax + by + c = 0$$

$$\underline{x} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \lambda x \\ \lambda y \\ \lambda \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ w' \end{bmatrix}$$

$$\underline{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \qquad \underline{l}^T \underline{x} = 0$$

Eq of line in Projective space

The point $\mathbf{x} \in \mathbb{P}^2$ lies on a line $\mathbf{l}$ if and only if $\mathbf{x}^\top \mathbf{l} = 0$.

Points are rays and lines are planes

$\in \mathbb{P}^2$, $\in \mathbb{R}^3$, $\in \mathbb{P}^2$, $\in \mathbb{R}^3$

Image model

$\underline{x} \in \mathbb{P}^2$

$\mathbb{R}^3$

$\underbrace{\hat{\underline{n}}^T \underline{X} = 0}_{\text{plane}}$

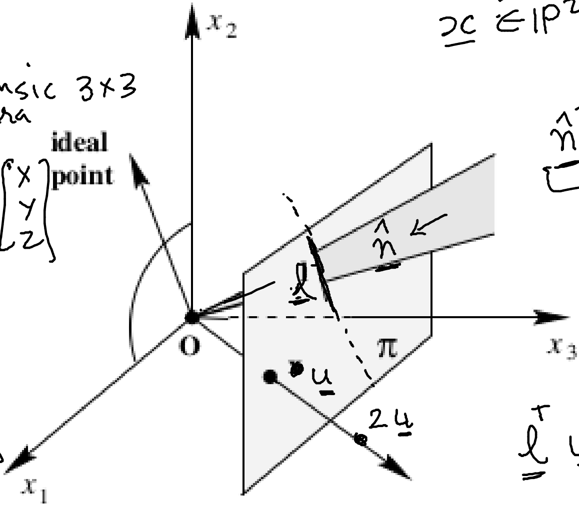
$\underline{u} = K \underline{X}$ — intrinsic 3×3 camera

$\underline{u} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$ $\quad \underline{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$

$\underline{u} \in \mathbb{P}^2$

$\underline{X} \in \mathbb{R}^3$

$\underline{u} = \begin{bmatrix} \lambda u \\ \lambda v \\ \lambda \end{bmatrix}$



$\underline{\underline{l}}^T \underline{u} = 0$

Intersection of lines

$$\underline{l}_1 = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} \qquad a_1 x + b_1 y + c_1 = 0$$

$$\underline{l}_2 = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix} \qquad a_2 x + b_2 y + c_2 = 0$$

Find a point $\underline{x}$ that lies on both $\underline{l}_1$ and $\underline{l}_2$

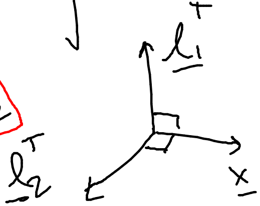
$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

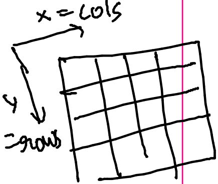
$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -c_1 \\ -c_2 \end{bmatrix}$$

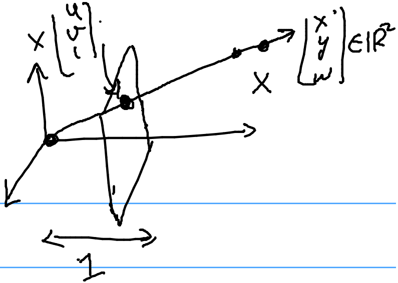
old way

$$\underline{l}_1^T \underline{x} = 0$$
$$\underline{l}_2^T \underline{x} = 0$$

$$\boxed{\underline{x} = \underline{l}_1 \times \underline{l}_2}$$







$$\underline{x} = \underline{l}_1 \times \underline{l}_2$$

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{vmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \frac{1}{w} \begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{vmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$\underline{u} = K \underline{X}$$

Intersection of parallel lines

Find the intersection of parallel lines $ax + by + c = 0$ ($\underline{l}_1$) and $ax + by + c' = 0$ ($\underline{l}_2$).

$$\underline{x} = \underline{l}_1 \times \underline{l}_2 = \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ a & b & c \\ a & b & c' \end{vmatrix}$$

$$= \begin{bmatrix} bc' - bc \\ -(ac' - ac) \\ ab - ab \end{bmatrix} = \begin{bmatrix} b \\ -a \\ 0 \end{bmatrix} (c' - c) =$$

$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{P}^2$ with $x_3 = 0$ then it is a point at ∞ $\quad \underline{x}_\infty$

Defn. Ideal point:

$\underline{x} \in \mathbb{P}^2 : x_3 = 0$

$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix}$ $\quad \underline{l}^T \underline{x} = 0 \quad \forall (x_1, x_2)$

$$\begin{bmatrix} l_1 & l_2 & l_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix} = 0$$

$$l_1 x_1 + l_2 x_2 + l_3 \cdot 0 = 0 \quad \forall x_1, x_2$$

$l_1 = 0, \ l_2 = 0$

$l_3 \neq 0$ $\quad$ $l_1 = l_2 = l_3 = 0$

$\notin \mathbb{P}^2$

$$\underline{l}_\infty = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$ line at infinity

all ideal points $\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix}$

lie on $\underline{l}_\infty = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Numerical example

line line $x = 1$

$x - 1 = 0$

Find the intersection of $x = 1$ and $y = 1$ using perspective geometry.

$ax + by + c = 0$

$y = 1$

$1x + 0 \cdot y - 1 = 0$

$0 \cdot x + 1 \cdot y - 1 = 0$

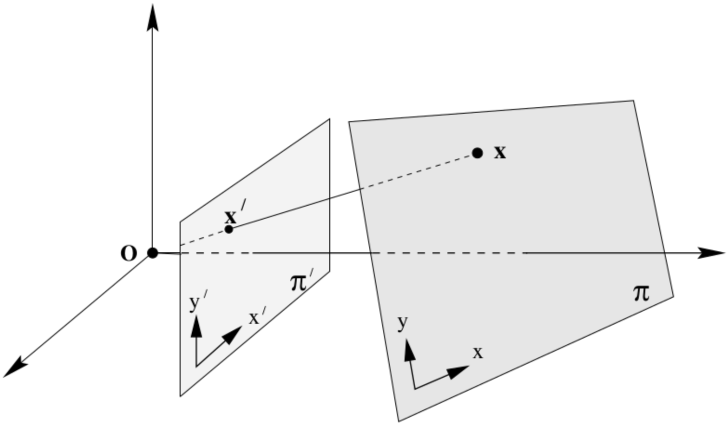
$\underline{l}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$\underline{l}_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

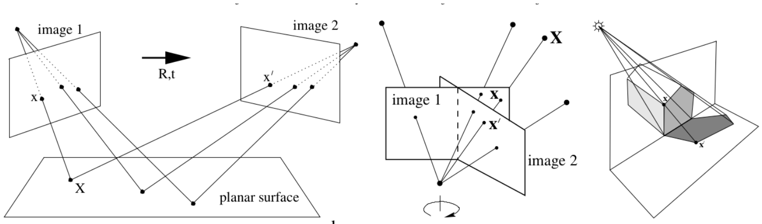
$\underline{x} = \underline{l}_1 \times \underline{l}_2$

Line joining points

Homography



Examples of Homography



a b c

Computing Homography


Top view
Graphic
Look! Safe to move?
Settings

2D homography

Given a set of points $\mathbf{x}_i \in \mathbb{P}^2$ and a corresponding set of points $\mathbf{x}'_i \in \mathbb{P}^2$, compute the projective transformation that takes each $\mathbf{x}_i$ to $\mathbf{x}'_i$. In a practical situation, the points $\mathbf{x}_i$ and $\mathbf{x}'_i$ are points in two images (or the same image), each image being considered as a projective plane $\mathbb{P}^2$.

Direct Linear Transformation (DLT) algorithm

Objective

Given $n \geq 4$ 2D to 2D point correspondences $\{\mathbf{x}_i \leftrightarrow \mathbf{x}'_i\}$, determine the 2D homography matrix $\mathtt{H}$ such that $\mathbf{x}'_i = \mathtt{H}\mathbf{x}_i$.

Algorithm

(i) For each correspondence $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$ compute the matrix $\mathtt{A}_i$ from (4.1). Only the first two rows need be used in general.

(ii) Assemble the n 2×9 matrices $\mathtt{A}_i$ into a single $2n \times 9$ matrix $\mathtt{A}$.

(iii) Obtain the SVD of $\mathtt{A}$ (section A4.4(*p*585)). The unit singular vector corresponding to the smallest singular value is the solution $\mathbf{h}$. Specifically, if $\mathtt{A} = \mathtt{UDV}^\mathsf{T}$ with $\mathtt{D}$ diagonal with positive diagonal entries, arranged in descending order down the diagonal, then $\mathbf{h}$ is the last column of $\mathtt{V}$.

(iv) The matrix $\mathtt{H}$ is determined from $\mathbf{h}$ as in (4.2).

3D to 2D camera projection matrix estimation

Given a set of points $\mathbf{X}_i$ in 3D space, and a set of corresponding points $\mathbf{x}_i$ in an image, find the 3D to 2D projective $\mathbf{P}$ mapping that maps $\mathbf{X}_i$ to $\mathbf{x}_i = \mathbf{P}\mathbf{X}_i$.

Eigenvalues and Eigenvectors

For a square matrix A, the λ_i and $\mathbf{x}_i$ that satisfy the following equation are called eigenvalues and eigenvectors respectively.

$$A\mathbf{x} = \lambda\mathbf{x} \text{ or } (A - \lambda I)\mathbf{x} = 0 \tag{3}$$

λ is chosen to ensure that $A - \lambda I$ has null space, hence, characteristic equation

$$\det(A - \lambda I) = 0 \tag{4}$$

For symmetrix matrix $A = A^\top$, eigenvalues are real, and eigenvectors are orthonormal,

$$A[\mathbf{x}_1, \dots, \mathbf{x}_n] = [\mathbf{x}_1, \dots, \mathbf{x}_n] \begin{bmatrix} \lambda_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \lambda_n \end{bmatrix} \tag{5}$$

$$AS = S\Lambda \tag{6}$$

$$\text{if } A = A^\top \text{ then } A = S\Lambda S^\top \tag{7}$$

Singular Value Decomposition (SVD)

$$A = U \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} V^\top \tag{8}$$

$$A^\top A = V \Sigma^2 V^{-1} \tag{9}$$

$$A^\top A \mathbf{v}_i = \lambda_i \mathbf{v}_i \qquad \lambda_i = \sigma_i^2 \tag{10}$$

$$AV = U \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} \tag{11}$$

$$U^+ = \Sigma^{-1} A V^+ \tag{12}$$