ECE 417/598: Camera calibration

Vikas Dhiman.

Feb 14, 2022

[1]

[1] See Hartley and Zisserman's Multiple View Geometry for details.

Announcements

- Midterm this Friday.
- Will cover
  - Rotation representations: Euler angles, axis-angle representation
  - Transformations
  - Denavit-Hartenberg parameters
  - Camera projection model
  - Pseudo-inverse
  - Projective representation for line and point
- Sample exam today
- No programming questions. Expect Linear Algebra question though. Only the concepts that we have touched in class.
- Jetbot update: probably not happening due to global chip shortage

$$u \approxeq v(\epsilon) \equiv |u - v| \leq \epsilon \min(|u|, |v|) \tag{1}$$

$$u \sim v(\epsilon) \equiv |u - v| \leq \epsilon \max(|u|, |v|) \tag{2}$$

Homogeneous representation of lines

$$\mathbb{P}^2 = \mathbb{R}^3 - \{(0,0,0)^\top\}$$

$$ax + by + 1.c = 0$$

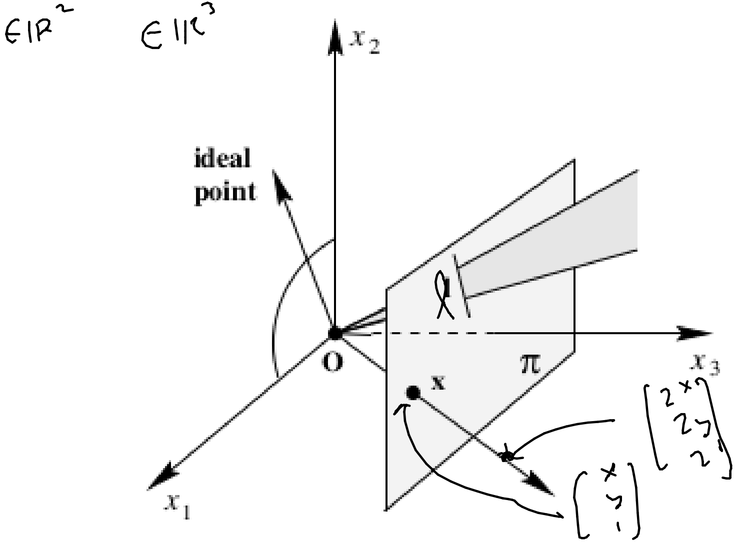
$$\mathbf{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \equiv \text{line}$$

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \lambda x \\ \lambda y \\ \lambda \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ w' \end{bmatrix}$$

The point $\mathbf{x} \in \mathbb{P}^2$ lies on a line $\mathbf{l}$ if and only if

$$\mathbf{l}^\top \mathbf{x} = 0$$

Points are rays and lines are planes



Intersection of lines

Two line $\mathbf{l}_1$ and $\mathbf{l}_2$ intersect at $\mathbf{x} \in \mathbb{P}^2$

$$\mathbf{x} = \mathbf{l}_1 \times \mathbf{l}_2$$

$$\underline{l}_1^T \underline{x} = 0$$

$$\underline{l}_2^T \underline{x} = 0$$

Numerical example

Find the intersection of two lines $x = 1$ and $y = 1$ using perspective geometry.

$x = 1$

$1 \cdot x + 0 \cdot y - 1 = 0$

$a x + b y + 1 \cdot c = 0$

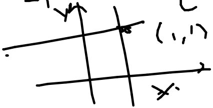
$l = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$y = 1$

$0 \cdot x + 1 \cdot y - 1 = 0$

$l = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

$$x = \begin{vmatrix} i & -j & k \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{vmatrix} = \begin{bmatrix} 0 - (-1) \\ -(-1 - 0) \\ 1 - 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \in \mathbb{P}^2$$



Numerical example

Find the intersection of two lines $x = y$ and $x + y = 1$ using perspective geometry.

$$x = y$$
$$x - y + 0 = 0$$
$$ax + by + c = 0$$

$$x + y = 1$$
$$x + y - 1 = 0$$

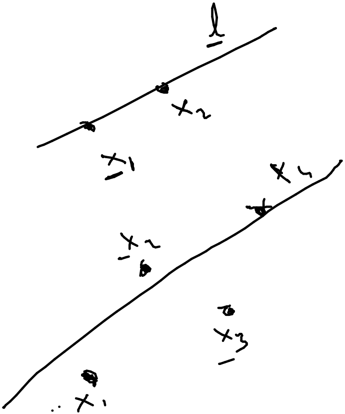
$$\underline{l}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$\underline{l}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$\underline{x} = \underline{l}_1 \times \underline{l}_2$$

Line joining points

Two point $\mathbf{x}_1$ and $\mathbf{x}_2$ form a $\mathbf{l} \in \mathbb{P}^2$



$$\mathbf{l} = \mathbf{x}_1 \times \mathbf{x}_2 \leftarrow \begin{cases} \mathbf{l}^T \mathbf{x}_1 = 0 \\ \mathbf{l}^T \mathbf{x}_2 = 0 \end{cases}$$

$\mathbf{x}_1 \in \mathbb{P}^2$

Overdetermined

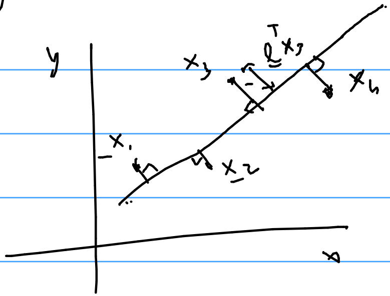
$\mathbf{l}^T \mathbf{x}_1 = 0$ $\quad$ $\mathbf{l}^T \mathbf{x}_3 = 0$

$\mathbf{l}^T \mathbf{x}_2 = 0$ $\quad$ $\mathbf{l}^T \mathbf{x}_4 = 0$

Least square solution

$\underline{l}^T x_i = 0$ for $i \in \{1, \dots n\}$

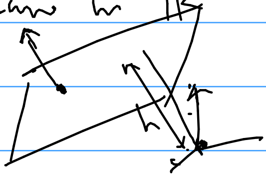
$\min_{\underline{l}} \sum_i \| l^T x_i \|^2$



line in $\mathbb{P}^2$ ⇒ plane in $\mathbb{R}^3$

$\hat{\underline{n}}^T \cdot \underline{x} = h$

↑ plane normal



$$\boldsymbol{l} = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix}$$

$$\min_{\boldsymbol{l}} \sum_i \| \boldsymbol{l}^T \boldsymbol{x}_i \|_2^2$$

$$\min_{\boldsymbol{l}} \left\| \begin{matrix} l^T x_1 \\ l^T x_2 \\ l^T x_3 \\ l^T x_n \end{matrix} \right\|_2^2$$

$$\|\boldsymbol{x}\|_2^2 = x_1^2 + x_2^2 + x_3^2$$

$$x_1^2 + x_2^2 + x_3^2$$

$$= \sum_{i \in 1,2,3} x_i^2 = \left\| \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} \right\|^2$$

$$\underline{x}^T \underline{y} = \underline{y}^T \underline{x}$$

$$\min_{\underline{l}} \left\| \underline{l}^T \overbrace{\begin{bmatrix} \underline{x}_1 & \underline{x}_2 & \underline{x}_3 & \underline{x}_4 \end{bmatrix}}^{A^T} \right\|_2^2$$

$$\min_{\underline{l}} \left\| \begin{matrix} \underline{x}_1^T \underline{l} \\ \underline{x}_2^T \underline{l} \\ \underline{x}_3^T \underline{l} \\ \underline{x}_4^T \underline{l} \end{matrix} \right\|_2^2 = \left\| \underbrace{\begin{bmatrix} \underline{x}_1^T \\ \underline{x}_2^T \\ \underline{x}_3^T \\ \underline{x}_4 \end{bmatrix}}_{A} \underline{l} \right\|_2^2$$

$$A_{4\times 3}$$

$$\underline{x} \in \mathbb{R}^2$$

$$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\min_{\underline{l}} \|A\underline{l}\|_2^2$$

$Ax - b$

$+ b = 0$

$x = A^{+} \underline{b}$

$$\left[\begin{array}{l} A\underline{l} = 0 \qquad\qquad = 0 \\ \underline{l} \in N(A) = \text{null space} \end{array} \right]$$

cover this soon

$$\underline{l} = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix}$$

$l_3 = 1$

$ax + by + c = 0$

$$= \begin{bmatrix} l_1 \\ l_2 \\ 1 \end{bmatrix}$$

$c = 1$

$c \neq 0$

$$A_{4\times 3}$$

$$\min_{[l_1, l_2]} \left\| A \begin{bmatrix} l_1 \\ l_2 \\ 1 \end{bmatrix} \right\|_2^2 \qquad \underline{y} = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}$$

$$\min_{\underline{y}} \left\| \left[A'_{4\times 2} \mid \underline{a}_{4\times 1} \right] \begin{bmatrix} \underline{y} \\ 1 \end{bmatrix} \right\|_2^2 \qquad A_{4\times 3} = \left[A'_{4\times 2} \mid \underline{a}_{4\times 1} \right]$$

$$\min_{\underline{y}} \left\| A'_{4\times 2}\underline{y} + \underline{a} \right\|_2^2 \qquad = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \\ x_4 & y_4 & 1 \end{bmatrix}$$

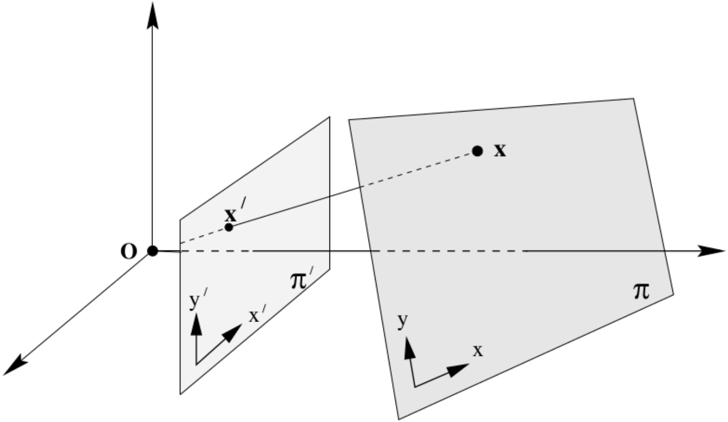
$$\underline{y} = -A'^{+}_{4\times 2}\,\underline{a}$$

$$A'^{+} = \left(A'^{T} A' \right)^{-1} A'^{T}$$

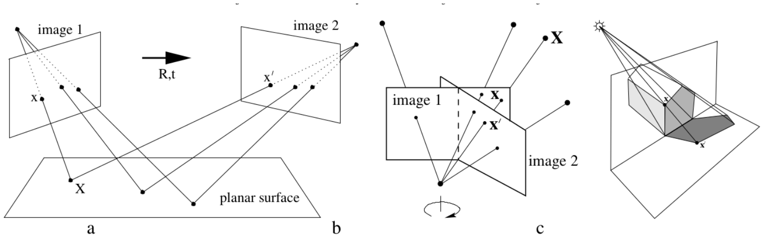
Numerical example

Find the lines that pass through points $(1, 1)$ and $(2, 2)$ using perspective geometry.

Homography



Examples of Homography




Top view
Graphic
Look! Safe to move?
Settings

Computing Homography

2D homography

Given a set of points $\mathbf{x}_i \in \mathbb{P}^2$ and a corresponding set of points $\mathbf{x}'_i \in \mathbb{P}^2$, compute the projective transformation that takes each $\mathbf{x}_i$ to $\mathbf{x}'_i$. In a practical situation, the points $\mathbf{x}_i$ and $\mathbf{x}'_i$ are points in two images (or the same image), each image being considered as a projective plane $\mathbb{P}^2$.

Computing Homography

Solving for Homography derivation

Direct Linear Transformation (DLT) algorithm

Objective

Given $n \geq 4$ 2D to 2D point correspondences $\{\mathbf{x}_i \leftrightarrow \mathbf{x}'_i\}$, determine the 2D homography matrix $\mathtt{H}$ such that $\mathbf{x}'_i = \mathtt{H}\mathbf{x}_i$.

Algorithm

(i) For each correspondence $\mathbf{x}_i \leftrightarrow \mathbf{x}'_i$ compute the matrix $\mathtt{A}_i$ from (4.1). Only the first two rows need be used in general.

(ii) Assemble the n 2×9 matrices $\mathtt{A}_i$ into a single $2n \times 9$ matrix $\mathtt{A}$.

(iii) Obtain the SVD of $\mathtt{A}$ (section A4.4(*p*585)). The unit singular vector corresponding to the smallest singular value is the solution $\mathbf{h}$. Specifically, if $\mathtt{A} = \mathtt{UDV}^{\mathsf{T}}$ with $\mathtt{D}$ diagonal with positive diagonal entries, arranged in descending order down the diagonal, then $\mathbf{h}$ is the last column of $\mathtt{V}$.

(iv) The matrix $\mathtt{H}$ is determined from $\mathbf{h}$ as in (4.2).

3D to 2D camera projection matrix estimation

Given a set of points $\mathbf{X}_i$ in 3D space, and a set of corresponding points $\mathbf{x}_i$ in an image, find the 3D to 2D projective $\mathbf{P}$ mapping that maps $\mathbf{X}_i$ to $\mathbf{x}_i = \mathbf{P}\mathbf{X}_i$.

Eigenvalues and Eigenvectors

For a square matrix A, the λ_i and $\mathbf{x}_i$ that satisfy the following equation are called eigenvalues and eigenvectors respectively.

$$A\mathbf{x} = \lambda\mathbf{x} \text{ or } (A - \lambda I)\mathbf{x} = 0 \tag{3}$$

λ is chosen to ensure that $A - \lambda I$ has null space, hence, characteristic equation

$$\det(A - \lambda I) = 0 \tag{4}$$

For symmetrix matrix $A = A^\top$, eigenvalues are real, and eigenvectors are orthonormal,

$$A[\mathbf{x}_1, \ldots, \mathbf{x}_n] = [\mathbf{x}_1, \ldots, \mathbf{x}_n] \begin{bmatrix} \lambda_1 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & \lambda_n \end{bmatrix} \tag{5}$$

$$AS = S\Lambda \tag{6}$$

$$\text{if } A = A^\top \text{ then } A = S\Lambda S^\top \tag{7}$$

Numerical example

Singular Value Decomposition (SVD)

$$A = U \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} V^\top \tag{8}$$

$$A^\top A = V \Sigma^2 V^{-1} \tag{9}$$

$$A^\top A \mathbf{v}_i = \lambda_i \mathbf{v}_i \qquad \lambda_i = \sigma_i^2 \tag{10}$$

$$AV = U \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} \tag{11}$$

$$U^+ = \Sigma^{-1} A V^+ \tag{12}$$

Numerical example