ECE 417/598: Review

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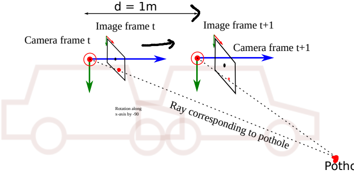
Concepts to review and remember

1. 2D/3D Rotation, translation, and transformation matrices
2. 3D Rotation from euler angles and vice versa
3. 3D Rotation from axis-angle and vice versa
4. Pinhole camera model. Image point to 3D ray and 3D point to image point. $X = \lambda K^{-1} \underline{y}$ $\underline{y} = \lambda K X$
5. Image line to 3D plane.
6. Least squares solution by function minimization $\min_x \|Ax - b\|^2$
7. Line-plane intersection
8. Line-line intersection
9. Plane-plane intersection ✓ → 3D line
10. SVD in terms of eigen values and vectors. Properties of eigen values, eigen vectors and SVD matrices.
11. Null space and column space } 4. fundamental space
12. Implicit and explicit equations of lines and planes.
13. Conversion of any linear system of equations into $Ax = b$ form or $Ax = 0$ form. ← null space

4

Find the 3D position of the pothole the $t+1$ coordinate frame, in terms of $d = 1$ (the movement of the camera), image-coordinates of the pothole $\underline{\mathbf{u}}_t$, $\underline{\mathbf{u}}_{t+1}$ (provided in figure), camera matrix K (provided in figure). The car has moved from directly forward along Z_t-axis by $d = 1$m without any rotation. We get two images at time t and at $t+1$. The detection of the pothole at time t is $\underline{\mathbf{u}}_t = [100, 75, 1]^\top$ and $\underline{\mathbf{u}}_{t+1} = [100, 95, 1]^\top$. Provide the formula or pseudo-code for computing the pothole coordinates.



⊙ Indicates out of the page towards viewer

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X-axis

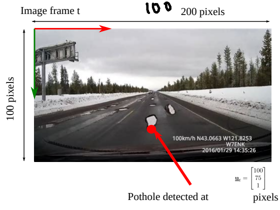
Z-axis

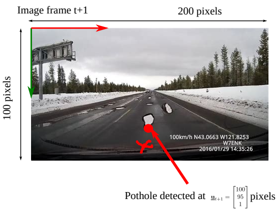
Y-axis

$$K = \begin{bmatrix} 100 & 0 & 100 \\ 0 & 100 & 50 \\ 0 & 0 & 1 \end{bmatrix}$$

line-line intersection

75





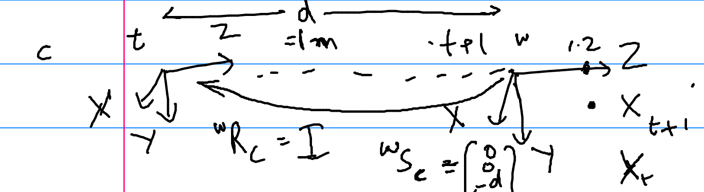
Step 1 : $X_t = \lambda_t K^{-1} \underline{u}_t$ ✓ $u_t = \begin{bmatrix} 100 \\ 75 \\ 1 \end{bmatrix}$

$\lambda_t \in \mathbb{R}$ unknown

✓ K = given

Step 2: $X_{t+1} = \lambda_{t+1} K^{-1} \underline{u}_{t+1}$

Step 3: Convert X_t to $t+1$ coordinate frame



$$x_t = \begin{bmatrix} 0 \\ 0 \\ 1.2 \end{bmatrix} \rightarrow X_{t+1} = ?$$

$$X_{t+1} = \begin{bmatrix} 0 \\ 0 \\ 1.2 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0.2 \end{bmatrix}$$

$$X_{t+1} = {}^{w}R_c X_t + {}^{w}s_c$$

$$= X_t + \begin{bmatrix} 0 \\ 0 \\ -d \end{bmatrix}$$

$$X_t = \lambda_t K^{-1} \underline{u}_t$$

$$X_{t+1} = \lambda_t K^{-1} \underline{u}_t + \begin{pmatrix} 0 \\ 0 \\ -d \end{pmatrix}$$

Step 4

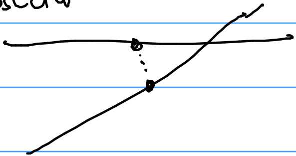
$$X_{t+1} = \lambda_{t+1} K^{-1} \underline{u}_{t+1}$$ line

$$X_{t+1} = \lambda_t K^{-1} \underline{u}_t + \begin{bmatrix} 0 \\ 0 \\ -d \end{bmatrix}$$ line

3D line-line intersection

Least.square

null space



$$\lambda_{t+1} K^{-1} \underline{u}_{t+1} = \lambda_t K^{-1} \underline{u}_t + \begin{bmatrix} 0 \\ 0 \\ -d \end{bmatrix}$$

$$\lambda_{t+1} K^{-1} \underline{u}_{t+1} - \lambda_t K^{-1} \underline{u}_t = \begin{bmatrix} 0 \\ 0 \\ -d \end{bmatrix}$$

$Ax=b$

$$3\ \underbrace{\overset{\longleftrightarrow\ 2}{\begin{bmatrix} K^{-1}u_{t+1} & -K^{-1}\underline{u}_t \end{bmatrix}}}_{A} \underbrace{\begin{bmatrix} \lambda_{t+1} \\ \lambda_t \end{bmatrix}}_{y} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ -d \end{bmatrix}}_{b}$$

3 eqns $\downarrow$ $\because$ of $\mathbb{P}^2$

$\boxed{\text{2 lin independent}}$

$$K_{3\times 3} \quad K^{-1}_{3\times 3} \quad \underline{u}_{t+1\ 3\times 1}$$

$$\left(K^{-1}u_{t+1}\right)_{3\times 1}$$

$$\underline{y} = \begin{bmatrix} \lambda_{t+1} \\ \lambda_t \end{bmatrix} = A^{\dagger} b = \left(A^T A\right)^{-1} A^T b$$

$$\left.\underline{X}_t = \lambda_{t+1} K^{-1} \underline{u}_{t+1}\ \right\}$$

point where the two lines intersect

Pseudo-inverse

$A^+ =$ through SVD

tall matrix $\quad A^+ = (A^T A)^{-1} A^T$

$A^T A$ to be invertible

$A^T_{n \times m} \; A_{m \times n}$ $\quad$ if A is full column rank

$(A^T A)_{n \times n}$ $\quad A \in \mathbb{R}^{m \times n}$

$\text{rank}(A) = n$

$$m \begin{array}{c} \xleftarrow{\;n\;} \rightarrow \\ \left[\quad \quad \quad \right] \end{array}$$

fat matrix $A^{+} = A^{T}(AA^{T})^{-1}b$

$\text{rank}(A) = m$

SVD : $A_{m \times n} = \underset{m \times m}{U}\ \overset{m \times n}{\Sigma}\ \underset{n \times n}{V^{T}}$

$$A = U \begin{bmatrix} \sigma_1 & & 0 & & \\ 0 & \ddots & \sigma_r & & 0 \\ & & & 0 & \\ & 0 & & & 0 \end{bmatrix} V^{T}$$

$$A^{+} = V \underbrace{\begin{bmatrix} 1/\sigma_1 & & & & \\ 0 & 1/\sigma_2 & 0 & & 0 \\ & & \ddots & 1/\sigma_r & \\ & 0 & & & 0 \end{bmatrix}}_{\Sigma^{-1}} U^{T}$$

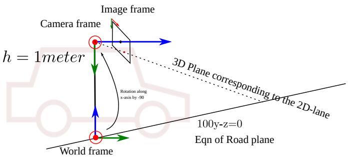
$$A^{+}A = \left(V\Sigma^{-1}U^{T}\right)\left(U\Sigma V^{T}\right)$$

$$= V\Sigma^{-1}\left(U^{T}U\right)\Sigma V^{T}$$

$$= V\Sigma^{-1} I_{m\times r}\Sigma V^{T}$$

$$= V\Sigma^{-1}\Sigma V^{T}$$

$$= V I V^{T} = V V^{T} = I$$



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$$K = \begin{bmatrix} 100 & 0 & 100 \\ 0 & 100 & 50 \\ 0 & 0 & 1 \end{bmatrix}$$

Lane detected at $\quad \mathbf{l} = \begin{bmatrix} 1 \\ -1 \\ -150 \end{bmatrix}$

In other words, the equation of the line detected in image coordinate frame is given by: $\quad (1)x + (-1)y + (-150)1 = 0$