ECE 417/598: Depth from stereo

Instructor: Vikas Dhiman

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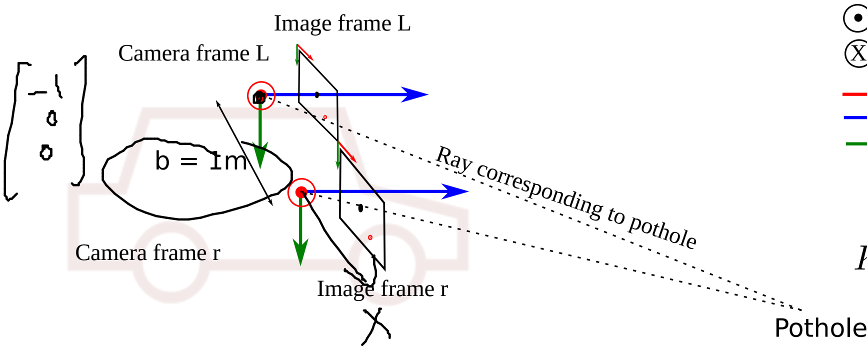
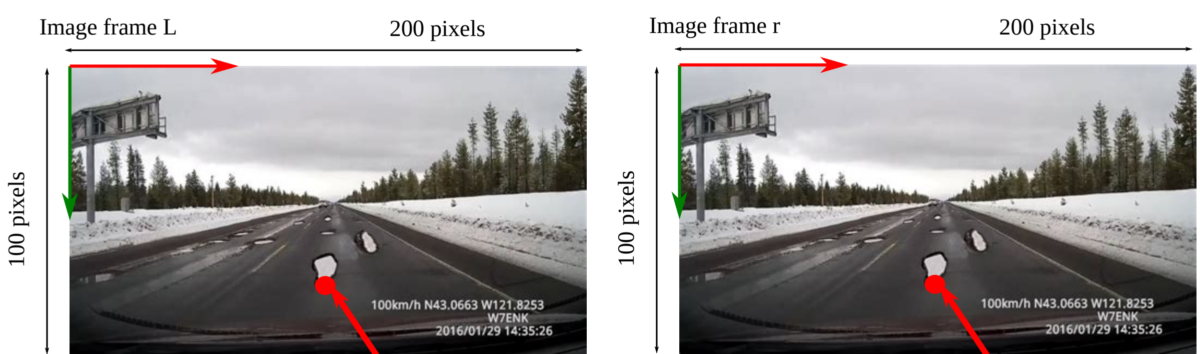
(1) Student name: Student email:

Problem 1 *Find the 3D position of the pothole give that the right camera r is moved along X-axis by $b = 1m$ with respect to the left camera L.*



⊙ Indicates out of the page towards viewer
ⓧ Indicates into the page away from viewer
X-axis
Z-axis
Y-axis

$$K = \begin{bmatrix} 100 & 0 & 100 \\ 0 & 100 & 50 \\ 0 & 0 & 1 \end{bmatrix}$$



Pothole detected at $\underline{\mathbf{u}}_L = \begin{bmatrix} 105 \\ 70 \\ 1 \end{bmatrix}$ pixels

Pothole detected at $\underline{\mathbf{u}}_r = \begin{bmatrix} 95 \\ 70 \\ 1 \end{bmatrix}$ pixels

1. Find the equation of ray corresponding to the pothole in the left image. In other words, write the equation of the ray corresponding to the point $\underline{\mathbf{u}}_L$, in camera frame L.

eqn of ray $\quad X_L = \lambda_L K^{-1} \underline{u}_L$

$$X_L = R^T X_r - R^T t$$

2. Find the rotation matrix ${}^rR_L$ and translation vector ${}^r\mathbf{t}_L$, so that they transform any point in the coordinate frame L to a point in the coordinate frame r. ($\mathbf{X}_r = {}^rR_L\mathbf{X}_L + {}^r\mathbf{t}_L$).

$${}^r\underline{R}_L = I \qquad {}^r\underline{t}_L = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -b \\ 0 \\ 0 \end{bmatrix}$$

3. Transform the equation of ray from coordinate frame L to coordinate frame r.

$$X_L = R\underline{X}_L + t$$

$$X_r = {}^rR_L(\lambda_L K^{-1} \underline{u}_L) + {}^r\underline{t}_L$$

$$X_r = \lambda_L ({}^rR_L K^{-1} \underline{u}_L) + {}^r\underline{t}_L$$

u_L 3×1

K^{-1} 3×3

${}^rR_L$ 3×3

4. Find the equation of ray corresponding to the pothole in the right image. In other words, write the equation of the ray corresponding to the point $\underline{\mathbf{u}}_r$, in camera frame r.

$$X_r = \lambda_r K^{-1} \underline{u}_r$$

5. Find the intersection of rays (lines) 4 and 3.

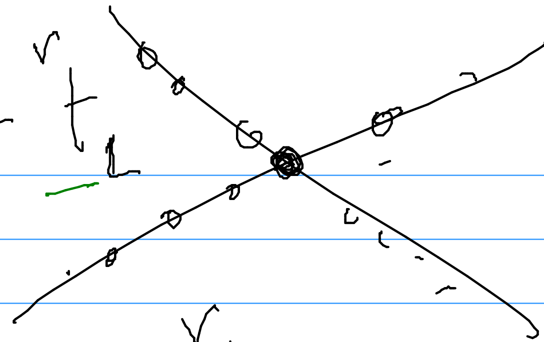
$$X_r = \lambda_L \underbrace{({}^rR_L K^{-1} \underline{u}_L)}_{d_1} + \underbrace{{}^r\underline{t}_L}_{3\times1}$$

$$X_r = \lambda_R \underbrace{K^{-1} \underline{u}_r}_{d_2}$$

$$x = \lambda_1 \underline{d}_1 + \underline{x}_1$$

$$x = \lambda_2 \underline{d}_2 + \underline{x}_2$$

$$\lambda_R K^{-1} \underline{u}_r = \lambda_L \left({}^rR_L K^{-1} \underline{u}_L\right) + {}^r\underline{t}_L$$



$$\lambda_R K^{-1} \underline{u}_r - \lambda_L \left({}^rR_L K^{-1} \underline{u}_L\right) = {}^r t_L$$

$$\overset{\longleftarrow\ 2\ \longrightarrow}{\underbrace{\begin{bmatrix} K^{-1}\underline{u}_r, & -{}^rR_L K^{-1}\underline{u}_L \end{bmatrix}}_{3\times 2}} \underbrace{\begin{bmatrix} \lambda_R \\ \lambda_L \end{bmatrix}}_{2\times 1} = \underbrace{{}^r\underline{t}_L}_{3\times 1}$$

$$\underline{A}\,\underline{x} = \underline{b}$$

$$\begin{bmatrix} \lambda_R \\ \lambda_L \end{bmatrix} = \underline{A}^{+}\,\underline{b}$$

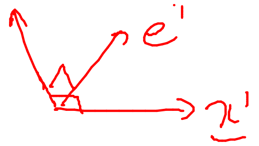
$$A^{+} = \left(A^T A\right)^{-1} A^T$$

3D point

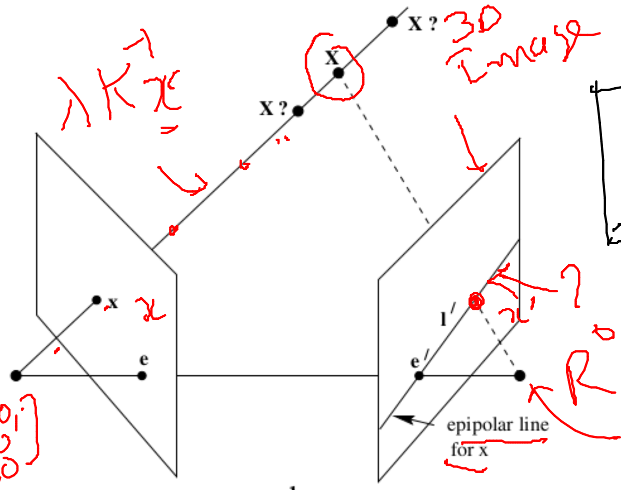
$$X_R = \lambda_R K_R^{-1} \underline{u}_R$$

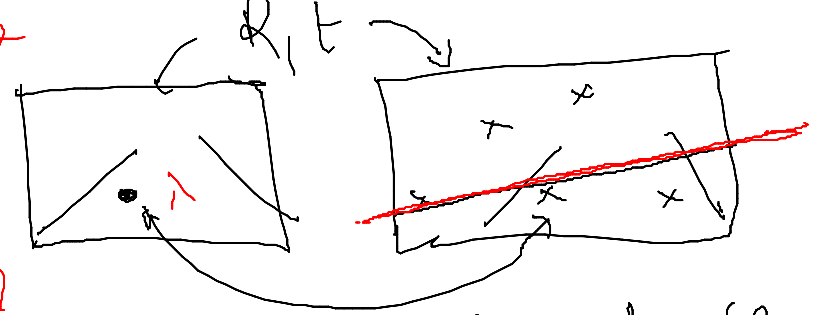
depth $= \underline{\underline{\lambda_R}}$

$$\lambda_R \begin{bmatrix} f & 0 & u_x \\ 0 & f & u_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ \\ 1 \end{bmatrix}$$



Problem 2 *When the depth corresponding to the point* $\mathbf{x}$ *is unknown, the possible pixels* ($\mathbf{x}'$) *on the right image that can correspond to the point form a line. What is the equation of that line?*





correspondance problem

$x = \begin{pmatrix}1\\2\\3\end{pmatrix}$

$e' = \begin{pmatrix}2\\4\\6\end{pmatrix}$

$\begin{cases} x'^T l' = 0 \\ e'^T l' = 0 \end{cases}$

$l' = e' \times x'$

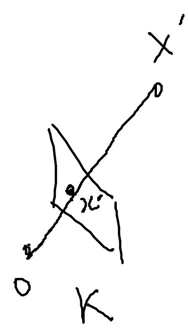
1. Find the equation of ray corresponding to the pothole in the left image. In other words, write the equation of the ray corresponding to the point $\mathbf{x}$, in the first camera frame.

$$X = \lambda K^{-1} x$$

2. Assume the rotation matrix R and translation vector $\mathbf{t}$, so that they transform any point in the left coordinate frame to a point in the right coordinate frame. ($\mathbf{X}' = R\mathbf{X} + \mathbf{t}$). Transform the equation of ray from the left coordinate frame to the right coordinate frame.

$$X' = R(\lambda K^{-1} x) + t$$

$$X' = \lambda (R K^{-1} x) + t$$



3. Project any 3D point on the right camera, call it $\mathbf{x}'$.

$$x' = \lambda' K X'$$

$$x' = \lambda' K [\lambda (R K^{-1} x) + t] = \lambda' \lambda K R K^{-1} x + \lambda' K t$$

4. Find the point $\mathbf{e}'$ on the right camera. imag

$$X'_0 = R \begin{bmatrix}0\\0\\0\end{bmatrix} + t = t$$

$$e' = \lambda'_e K X'_0 = \lambda'_e K t$$

$$e'^T l' = 0$$
$$l' = e' \times x' \qquad x'^T l' = 0$$
$$= (\lambda_e' K t) \times (\lambda' \lambda R K^{-1} \underline{x} + \lambda' K t)$$
$$= \lambda_e' \lambda' \left[(Kt) \times (\lambda R K^{-1} x) + \underbrace{(Kt) \times (Kt)}_{0} \right]$$
$$= \lambda_e' \lambda' \lambda (Kt) \times (R K^{-1} x)$$
$$l' = (Kt) \times (R K^{-1} \underline{x})$$

5. Find a line $\mathbf{l}'$ that passes through both $\mathbf{x}'$ and $\mathbf{e}'$.